Multiuser Multihop MIMO Relay System Design Based on Mutual Information Maximization

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Abstract—In this paper, we consider multiuser multihop relay communication systems, where the users, relays, and the destination node may have multiple antennas. We address the issue of source and relay precoding matrices design to maximize the system mutual information (MI). By exploiting the link between the maximal MI and the weighted minimal mean-squared error (WMMSE) objective functions, we show that the intractable maximal MI-based source and relay optimization problem can be solved via the WMMSE-based source and relay design through an iterative approach which is guaranteed to converge to at least a stationary point. For the WMMSE problem, we derive the optimal structure of the relay precoding matrices and show that the WMMSE matrix at the destination node can be decomposed into the sum of WMMSE matrices at all hops. Under a (moderately) high signal-to-noise ratio (SNR) condition, this WMMSE matrix decomposition significantly simplifies the solution to the WMMSE problem. Numerical simulations are performed to demonstrate the effectiveness of the proposed algorithm.

Index Terms—MIMO relay, multiuser, multihop relay, mutual information.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) relay communication technique has attracted much research interest due to its capability in enhancing the system reliability and extending the network coverage [1]-[3]. The relay node can use regenerative or non-regenerative relay strategies [4]. As the distance between source and destination increases, in order to guarantee the system coverage, multiple relay nodes are needed to relay signals from source to destination. In such scenario, non-regenerative MIMO relay systems have been shown to outperform the regenerative ones in computational complexity and system delay [5].

For a single-user multihop MIMO relay system with any number of hops, the optimality of channel diagonalization has been proven in [6]. For a downlink multiuser MIMO relay system where each user is equipped with a single antenna, the source and relay precoding matrices design has been investigated in [7]-[10]. In particular, the upper and lower bounds of the achievable sum rate have been established in [7]. Source and relay matrices that maximize the sum capacity have been studied in [8]. A joint beamforming and power allocation algorithm has been developed in [9] considering the quality-of-service (QoS) constraints. In [10], the mismatch between the true and outdated channel state information (CSI) has been considered in the transceiver design.

In multiuser two-hop relay systems, where the users and the relay node are equipped with multiple antennas, the source and relay precoding matrices maximizing the system mutual information (MI) have been derived in [11] and [12]. In particular, the sum rate maximization and the power minimization problems have been studied in [11], while the weighted sum rate maximization has been considered in [12]. Transceiver designs for two-hop interference MIMO relay systems have been addressed in [13] and [14]. In the multiuser multihop MIMO relay uplink communication system, a simplified algorithm of optimizing the source and relay precoding matrices based on the minimal mean-squared error (MMSE) criterion has been proposed in [15].

In this paper, we focus on multiuser multihop linear non-regenerative (amplify-and-forward) MIMO relay communication systems. Different to the MMSE objective in [15], we aim at maximizing the system MI. The MI maximization problem is more challenging to solve than the MMSE optimization problem. Compared with [7]-[14] which consider only two-hop relay systems, we address multihop multiuser relay systems with any number of hops. Since the MI-based source and relay matrices design problem is intractable to solve, we convert the original problem to weighted MMSE (WMMSE)-based problem by exploiting the link between the MI and WMMSE objectives. We would like to mention that such link was first established for a single-hop MIMO system in [16]. Later on, it has been used in transceiver design for interference MIMO systems [17]. In this paper, we extend the MI-WMMSE link to multihop multiuser MIMO relay systems with any number of hops.

We develop an iterative algorithm to maximize the system MI by solving the WMMSE problem at each iteration. We prove that this iterative procedure is guaranteed to converge to a stationary point. For the WMMSE problem, we derive the structure of the optimal relay precoding matrices and show that the WMMSE matrix at the destination node can be decomposed into the sum of WMMSE matrices at all hops. We would like to mention that the decomposition of the (un-weighted) MMSE matrix was first discovered in [18] for a single-user two-hop MIMO relay system, and was extended to multiuser multihop MIMO relay systems in [15]. Our paper generalizes [15] from un-weighted MMSE matrix decomposition to WMMSE matrix decomposition in multiuser
multihop MIMO relay systems with any number of hops and any number of users.

At a (moderately) high signal-to-noise ratio (SNR), the WMMSE matrix decomposition enables the overall WMMSE optimization problem to be decomposed into subproblems of the source precoding matrices optimization and the relay precoding matrices optimization. Such decomposition greatly reduces the complexity of solving the WMMSE problem. In this way, the relay precoding matrices can be optimized successively with the local channel state information (CSI) knowledge and the weight matrix in each iteration. Moreover, we find that the WMMSE problem for optimizing the source precoding matrices is more challenging to solve than the relay precoding matrices optimization problem in [15]. Interestingly, we show that this subproblem can be transformed into the WMMSE-based joint transmitter and receiver optimization of a single-hop multiuser MIMO uplink communication system.

An iterative algorithm is developed to solve this equivalent problem by updating the transmitter precoding matrices and the receiver matrix alternately. In particular, each source precoding matrix can be updated independently using the Lagrange multiplier method.

We would like to note that the algorithms in [20] can be used to solve a general class of non-convex optimization problems in multihop MIMO relay networks including the weighted sum-rate maximization. However, our proposed algorithm can exploit the optimal structure of the relay precoding matrices (in Theorem 2) to reduce the complexity and improve the convergence rate for the special case of sum-rate maximization.

Numerical simulations demonstrate the effectiveness of the proposed algorithms, which typically converge in a few iterations. We would like to note that although we focus on multiaccess MIMO relay systems, the algorithms developed in this paper can be applied to broadcasting MIMO relay systems by exploiting the uplink-downlink duality for multihop linear non-regenerative MIMO relay systems [19]-[22]. For notational convenience, we consider a narrow band single-carrier system in this paper, and our algorithm can be applied in each subcarrier of a broadband multicarrier multihop MIMO relay system.

The rest of the paper is organized as follows. In Section II, we present the model of a linear non-regenerative multiuser multihop MIMO relay communication system. The proposed source and relay precoding matrices design algorithms are developed in Section III. In Section IV, numerical examples are shown to demonstrate the performance of the proposed algorithms. Conclusions are drawn in Section V. The following notations are used throughout the paper: $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, $\text{tr}(\cdot)$, $|\cdot|$ denote the matrix transpose, Hermitian transpose, inversion, trace, and determinant, respectively; $E[\cdot]$ stands for the statistical expectation with respect to the signal and noise; $bd(\cdot)$ denotes a block diagonal matrix; $I_n$ denotes the $n \times n$ identity matrix.

II. SYSTEM MODEL AND MAIN OBJECTIVE

We consider a multiuser multihop MIMO relay communication system as shown in Fig. 1, where $N_u$ users simultaneously transmit information to one destination node via $L - 1$ relay nodes in serial. The $l$th relay node has $N_l$, $l = 1, \cdots, L - 1$, antennas, and the destination node has $N_L$ antennas. The $i$th user is equipped with $M_i$, $i = 1, \cdots, N_u$, antennas. The total number of independent data streams from all users is denoted as $N_0 = \sum_{i=1}^{N_u} M_i$, which should satisfy $N_0 \leq \min\{N_1, \cdots, N_L\}$, so that the system can support $N_0$ active symbols in each transmission. We assume the orthogonality among different hops, as adopted in [6], [20], and [21], meaning that the signal transmitted by the $l$th relay can only be received by the $(l + 1)$-th relay due to the propagation pathloss and proper channel reuse.

The $M_1 \times 1$ source signal vector $s_i$ is linearly precoded by the $M_1 \times M_i$ source precoding matrix $B_i$. The precoded signal vectors

$$u_i = B_is_i, \quad i = 1, \cdots, N_u$$

are transmitted to the first relay node. The received signal vector at the first relay node is given by

$$y_1 = \sum_{i=1}^{N_u} G_i B_is_i + v_1 \triangleq H_1 F_1 s + v_1 \triangleq H_1 x_1 + v_1$$

where $G_i$ is the $N_1 \times M_i$ MIMO channel matrix between the $i$th user and the first relay node, $v_1$ is the $N_1 \times 1$ independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) vector at the first relay node, $s \triangleq [s_1^T, \cdots, s_{N_u}^T]^T$ is the vector of all source signals, $x_1 = F_1 s$ is the signal vector transmitted by all source nodes, and

$$H_1 \triangleq [G_1, \cdots, G_{N_u}], \quad F_1 \triangleq bd(B_1, \cdots, B_{N_u}).$$

Here we assumed that all users are synchronized perfectly. In (3), $H_1$ stands for the equivalent $N_1 \times N_0$ channel matrix between all users and the first relay node, and $F_1$ stands for the $N_0 \times N_0$ block diagonal source precoding matrix of all users. We assume that $E[ss^H] = I_{N_0}$.

We adopt the non-regenerative relay strategy as in [6], where each relay node amplifies (linearly precodes) and forwards its received signals. Thus, the relationship between the input and output vectors at the $l$th relay node is given by

$$x_{l+1} = F_{l+1} y_l, \quad l = 1, \cdots, L - 1$$

where $F_{l+1}$ is the $N_l \times N_l$ precoding matrix at the $l$th relay node, and $y_l$ is the $N_l \times 1$ signal vector received by the $l$th relay node with

$$y_l = H_l x_l + v_l, \quad l = 1, \cdots, L - 1.$$
Here $H_l$ is the $N_l \times N_{l-1}$ channel matrix of the $l$th hop, and $v_l$ is the $N_l \times 1$ i.i.d. AWGN vector at the $l$th relay node. The signal vector received at the destination node is given by (5) with $l = L$. We assume that all noises are complex circularly symmetric with zero mean and unit variance.

From (2)-(5), we have

$$y_l = A_l s + v_l, \quad l = 1, \cdots, L$$

(6)

where $A_l$ is the equivalent MIMO channel matrix given by

$$A_l = \prod_{i=l}^{1} (H_i F_i), \quad l = 1, \cdots, L$$

(7)

and $v_l$ is the equivalent noise vector whose covariance matrix is

$$C_1 = I_{N_1}$$

$$C_l = \sum_{j=2}^{l} \left( \prod_{i=j}^{l} (H_i F_i) \prod_{i=j}^{l} (F_i^H H_i^H) \right) + I_{N_1}, \quad l = 2, \cdots, L.$$ From (1), the transmission power at the $i$th user is $tr (B_i B_i^H), i = 1, \cdots, N_u$. Using (4) and (5), the transmission power consumed by the $(l - 1)$-th relay node can be written as

$$tr (E[x_l x_l^H]) = tr (F_l D_{l-1} F_l^H), \quad l = 2, \cdots, L$$

(8)

where

$$D_l \triangleq E[y_l y_l^H] = A_l A_l^H + C_l, \quad l = 1, \cdots, L - 1$$

(9)

is the covariance matrix of $y_l$.

Our main objective is to find the optimal source precoding matrices $\{B_i\} \triangleq \{B_1, \cdots, B_{N_u}\}$ and relay precoding matrices $\{F_i\} \triangleq \{F_2, \cdots, F_L\}$ to maximize the system MI $[23]$, [6] subjecting to transmission power constraint at the users and the relay nodes, which can be written as

$$\max_{\{B_i\}, \{F_i\}} \log |I_{N_0} + A_L^H C_L^{-1} A_L|$$

(10)

$$\text{s.t.} \quad \text{tr}(F_l D_{l-1} F_l^H) \leq p_l, \quad l = 2, \cdots, L$$

(11)

$$\text{tr}(B_i B_i^H) \leq q_i, \quad i = 1, \cdots, N_u$$

(12)

where $p_l$ is the power available at the $(l - 1)$-th relay node and $q_i$ is the power budget at the $i$th user.

The problem (10)-(12) is highly non-convex with matrix variables. It is computationally intractable to obtain the globally optimal solution, in particular for multihop systems with $L \geq 3$. In the following, we propose simplified algorithms with low computational complexity for the problem (10)-(12) by exploiting the MSE matrix decomposition technique [15], and the link between the maximal MI and the WMMSE objectives [16].

III. PROPOSED SOURCE AND RELAY PRECODING MATRICES DESIGN ALGORITHMS

Let us introduce the MMSE matrix $E_L$ of the signal waveform estimation at the destination node as [6], [15]

$$E_L = (I_{N_0} + A_L^H C_L^{-1} A_L)^{-1}.$$ (13)

We now show that the link between the WMMSE and maximal MI objectives in a single-hop MIMO system established in [16] can be extended to multiuser multihop MIMO relay systems.

THEOREM 1: By introducing a Hermitian weight matrix $W$, the problem (10)-(12) has the same first order optimality condition as the following problem

$$\min_{\{B_i\}, \{F_i\}, W} \text{tr}(W(I_{N_0} + A_L^H C_L^{-1} A_L)^{-1}) - \log |W|$$

(14)

$$\text{s.t.} \quad \text{tr}(F_l D_{l-1} F_l^H) \leq p_l, \quad l = 2, \cdots, L$$

(15)

$$\text{tr}(B_i B_i^H) \leq q_i, \quad i = 1, \cdots, N_u$$

(16)

when

$$W = E_L^{-1}.$$ (17)

Moreover, with given $\{B_i\}$ and $\{F_i\}$, the weight matrix $W$ minimizing (14) is given by (17).

PROOF: See Appendix A.

Based on Theorem 1, we propose an iterative algorithm for the problem (10)-(12), where in each iteration, with $W$ from the previous iteration, we first optimize $\{B_i\}$ and $\{F_i\}$ through solving the WMMSE problem (14)-(16). Then, we update $W$ as (17) using $\{B_i\}$ and $\{F_i\}$ obtained in the current iteration. Note that the conditional updates of $\{B_i\}$, $\{F_i\}$ and $W$ may either decrease or maintain but cannot increase the objective function (14). Monotonic convergence of the iterative algorithm towards (at least) a stationary point follows directly from this observation.

A. Decomposition of the WMMSE Matrix

With fixed $W$, the second term in (14) is constant. Thus, the problem (14)-(16) can be rewritten as the following WMMSE problem

$$\min_{\{B_i\}, \{F_i\}} \text{tr}(W^T E_L W^T)$$

(18)

$$\text{s.t.} \quad \text{tr}(F_l D_{l-1} F_l^H) \leq p_l, \quad l = 2, \cdots, L$$

(19)

$$\text{tr}(B_i B_i^H) \leq q_i, \quad i = 1, \cdots, N_u$$

(20)

where $W = W^* W^T$ and $W^* = W^{T/2}$. The problem (18)-(20) is non-convex with matrix variables. A globally optimal solution is very difficult to obtain with reasonable computational complexity. However, the WMMSE matrix $E_L \triangleq W^* E_L W^T$ can be decomposed into $L$ MMSE matrices as shown below.

THEOREM 2: By introducing $N_{l-1} \times N_0$ matrices $T_l, l = 2, \cdots, L$, the optimal $\{F_i\}$ as the solution to the problem (18)-(20) can be written as

$$F_l = T_l W^* A_L^H D_{l-1}^{-1}, \quad l = 2, \cdots, L.$$ (21)
With (21), $\tilde{E}_L$ can be decomposed to
\[
\tilde{E}_L = W^H (I_{N_0} + F_1^H H_1^H F_1)^{-1} W^T + \sum_{l=2}^{L} (T_l^H H_l^T H_l + R_l^{-1})^{-1}
\]
where
\[
R_l = W^H A_{l-1}^T D_{l-1} A_{l-1} W^2, \quad l = 2, \ldots, L. \tag{23}
\]
In (21), $\{T_l\} \triangleq \{T_2, \ldots, T_L\}$ is the optimal solution to the following problem
\[
\min_{\{B_l\}, \{T_l\}} \text{tr} \left( W^H (I_{N_0} + F_1^H H_1^H F_1)^{-1} W^T + \sum_{l=2}^{L} (T_l^H H_l^T H_l + R_l^{-1})^{-1} \right) \tag{24}
\]
s.t. $\text{tr}(T_l R_l T_l^H) \leq p_l, \quad l = 2, \ldots, L \tag{25}
\]
\[
\text{tr}(B_l B_l^H) \leq q_i, \quad i = 1, \ldots, N_u. \tag{26}
\]
**PROOF:** See Appendix B.

We would like to note that the MMSE matrix decomposition for multihop MIMO relay systems has been discovered in [15] when $W$ is an identity matrix. Therefore, Theorem 2 extends the result in [15] to the general case of $W \neq I_{N_0}$.

Using the matrix inversion lemma
\[
(A + BCD)^{-1} = A^{-1} - A^{-1} B (DA^{-1} B + C^{-1})^{-1} DA^{-1}
\]
we can rewrite $R_l$, $l = 2, \ldots, L$, as
\[
R_l = W^H A_{l-1}^T C_{l-1}^{-1} A_{l-1} (A_{l-1}^T C_{l-1}^{-1} A_{l-1} + I_{N_0})^{-1} W^T.
\]
In the case of (moderately) high SNR where $A_{l-1}^T C_{l-1}^{-1} A_{l-1} \gg I_{N_0}$, $l = 2, \ldots, L$, we have $R_l \approx W$, $l = 2, \ldots, L$. This indicates that in this case, $\{B_l\}$ and $\{T_l\}$ have almost no impact on $R_l$, $l = 2, \ldots, L$, which implies that the objective function (24) and the constraints in (25) are decoupled with respect to the variables $\{B_l\}$ and $\{T_l\}$. Thus, the problem (24)-(26) can be approximated and decomposed into the source precoding matrices optimization problem
\[
\min_{\{B_l\}} \text{tr} \left( W^H (I_{N_0} + F_1^H H_1^H F_1)^{-1} W^T \right) \tag{28}
\]
s.t. $\text{tr}(B_l B_l^H) \leq q_i, \quad i = 1, \ldots, N_u \tag{29}
\]
and the relay precoding matrix optimization problem for each $T_l$, $l = 2, \ldots, L$
\[
\min_{T_l} \text{tr} \left( (T_l^H H_l^T H_l T_l + R_l^{-1})^{-1} \right) \tag{30}
\]
s.t. $\text{tr}(T_l R_l T_l^H) \leq p_l. \tag{31}
\]
In the next two subsections, we focus on solving the problem (28)-(29) and the problem (30)-(31).

### B. The Source Matrices Optimization

When $W = I_{N_0}$, it is shown in [15] that the problem (28)-(29) can be converted to a convex semidefinite programming (SDP) problem. However, for general $W \neq I_{N_0}$, the problem (28)-(29) cannot be cast as a convex optimization problem. Interestingly, as (28) is the WMMSE of the single-hop multiuser MIMO system (2), it can be written as
\[
\text{tr} \left( W^H (I_{N_0} + F_1^H H_1^H F_1)^{-1} W^T \right) = \min \text{tr} \left( W E[L^H y_1 - s] (L^H y_1 - s)^H \right) \quad \text{L}
\]
where $L$ is the weight matrix of the linear receiver for the MIMO system in (2). To see this, let us work out the expectation on the right-hand side of (32) as
\[
\text{tr} \left( W E[(L^H y_1 - s) (L^H y_1 - s)^H] \right) = \text{tr}(WLH^T (H_1 F_1 F_1^H H_1^H + I_{N_0}) L - L^H H_1 F_1 - F_1^H H_1^H L + I_{N_0}). \tag{33}
\]
The optimal $L$ minimizing (33) is the Wiener filter [25] given by
\[
L = (H_1 F_1 F_1^H H_1^H + I_{N_1})^{-1} H_1 F_1. \tag{34}
\]
By substituting (34) back to (33), we obtain the left-hand side of (32).

By exploiting (32), the problem (28)-(29) can be solved via the following problem
\[
\min \text{tr} \left( (W^H L H_1 F_1 - W^H) (W^H L H_1 F_1 - W^H)^H \right) + W^H L W \tag{35}
\]
s.t. $\text{tr}(B_l B_l^H) \leq q_i, \quad i = 1, \ldots, N_u. \tag{36}
\]
In the following, we propose an iterative algorithm for the problem (35)-(36). In each iteration, we first optimize $L$ as given by (34) based on $\{B_l\}$ from the previous iteration. Then using $L$ obtained in the current iteration, we optimize $\{B_l\}$ by solving the problem of
\[
\min_{\{B_l\}} \text{tr} \left( (Z F_1 - W^H) (Z F_1 - W^H)^H \right) \tag{37}
\]
s.t. $\text{tr}(B_l B_l^H) \leq q_i, \quad i = 1, \ldots, N_u \tag{38}
\]
where $Z \triangleq W^H L^H H_1$. We update $\{B_l\}$ and $L$ alternatingly till convergence.

Let us introduce $Z_l$ and $W_l$ which contain the $\sum_{j=0}^{M_j-1} M_j$ columns of $Z$ and $W^H$ respectively, $i = 1, \ldots, N_u$, where $M_0 = 0$. We can rewrite (37) as
\[
\sum_{i=1}^{N_u} \sum_{l=1}^{N_u} \text{tr} \left( (Z_l B_l - W_l) (Z_l B_l - W_l)^H \right). \tag{39}
\]
It can be seen from (38) and (39) that the problem (37)-(38) can be decomposed into $N_u$ subproblems, where each $B_i$ is optimized through solving the following problem
\[
\min_{B_i} \text{tr} \left( (Z_l B_l - W_l) (Z_l B_l - W_l)^H \right) \tag{40}
\]
s.t. $\text{tr}(B_l B_l^H) \leq q_i. \tag{41}
\]
Using the Lagrange multiplier method [26], the solution to the problem (40)-(41) is given by
\[
B_i = (Z_l^T Z_l + \lambda_i I_{M_i})^{-1} Z_l^T W_l, \quad i = 1, \ldots, N_u \tag{42}
\]
where $\lambda_i \geq 0$ is the Lagrangian multiplier and can be found by substituting (42) back into (41) and solve the obtained equation using the bisection search [26].
We would like to mention that the conditional updates of \{B_i\} and \(L\) may either decrease or maintain but cannot increase the objective function (35). Monotonic convergence of the source matrices optimization algorithm towards (at least) a stationary point follows directly from this observation.

C. The Relay Matrices Optimization

Let us introduce the eigenvalue decomposition (EVD) of \(H_l^H H_l = U_l \Lambda_l U_l^H\) and \(R_l = U_l \Sigma_l U_l^H\), \(l = 2, \ldots, L\), where \(\Lambda_l\) and \(V_l \Sigma_l \) are \(N_l \times N_l\) matrices, the dimensions of \(U_l\) and \(\Sigma_l\) are \(N_l \times N_0\), and the diagonal elements of \(\Lambda_l\) and \(\Sigma_l\) are both sorted in descending order. It can be shown using Lemma 2 in [15] that the solution to the relay matrices optimization problem (30)-(31) has a water-filling solution as

\[ T_l = V_{l,1} \Delta_l U_{l,1}^H, \quad l = 2, \ldots, L \]

(43)

where \(V_{l,1}\) denotes the leftmost \(N_0\) columns of \(V_l\), and \(\Delta_l\) is an \(N_0 \times N_0\) diagonal matrix that remains to be optimized. Based on (21) and (43), the relay matrices are given by

\[ F_l = V_{l,1} \Delta_l U_l^H W^2 A_{l-1}^H D_{l-1}^{-1}, \quad l = 2, \ldots, L. \]

(44)

Substituting (43) back into (30)-(31), we obtain the following optimal power loading problem with scalar variables

\[ \min_{\delta_{l,i}, \lambda_{l,i}, \sigma_{l,i}, \eta_{l,i}} \sum_{i=1}^{N_0} \frac{1}{\sigma_{l,i}^2 + \lambda_{l,i}^2} \quad \text{s.t.} \sum_{i=1}^{N_0} \delta_{l,i}^2 \sigma_{l,i} \leq P_l \]

(45)

(46)

where \(\delta_{l,i}, \lambda_{l,i}, \sigma_{l,i}, \eta_{l,i}, i = 1, \ldots, N_0\), denote the \(i\)th diagonal element of \(\Delta_l\), \(\Sigma_l\), \(\Lambda_l\), respectively. The problem (45)-(46) can be solved by the Lagrange multiplier method as

\[ \delta_{l,i}^2 = \frac{1}{\lambda_{l,i}} \left( \frac{\sqrt{\lambda_{l,i}^2 + \mu_l \sigma_{l,i}^2}}{\mu_l \sigma_{l,i}} - \frac{1}{\sigma_{l,i}} \right)^+ \quad i = 1, \ldots, N_0 \]

(47)

where \((x)^+ \triangleq \max(x, 0)\) and \(\mu_l > 0\) is the Lagrangian multiplier and the solution to the nonlinear equation

\[ \sum_{i=1}^{N_0} \sigma_{l,i} (\frac{\sqrt{\lambda_{l,i}^2 + \mu_l \sigma_{l,i}^2}}{\mu_l \sigma_{l,i}} - \frac{1}{\sigma_{l,i}})^+ = P_l. \]

D. Summary and Comments

The procedure of the proposed source and relay matrices design algorithm is summarized in Table I, where \(\varepsilon_1\) and \(\varepsilon_2\) are small positive numbers close to zero up to which convergence is acceptable, \(\max \cdot \) stands for the maximum of the absolute value of all elements in a matrix, and the superscript \((n)\) and \([m]\) denote the number of iterations at the outer loop and the inner loop, respectively.\(^2\)

The major operation in each iteration of the proposed algorithm involves matrix inversion and matrix EVD. Thus, the per-iteration computational complexity order of the proposed algorithm is \(O\left(\sum_{l=0}^{L-1} N_l^3\right)\). The overall complexity depends on the number of iterations till convergence. It will be shown in Section IV that the proposed algorithm converges usually in less than 10 iterations.

Interestingly, as \(R_l\) can be approximated as \(W\) at (moderately) high SNRs, the relay matrices optimization problem can be further simplified by substituting \(R_l\) in (30)-(31) with \(W\), which can be rewritten as

\[ \min_{T_l} \text{tr}(T_l^H H_l^H H_l T_l + W^{-1}) \]

(48)

\[ \text{s.t.} \text{tr}(T_l W T_l^H) \leq P_l. \]

(49)

By introducing the EVD of \(W = U_w \Sigma_w U_w^H\) and using Lemma 2 in [15], the solution to the problem (48)-(49) is given by

\[ T_l = V_{l,1} \Theta_l U_w^H, \quad l = 2, \ldots, L. \]

(50)

Based on (21) and (50), the relay matrices are given by

\[ F_l = V_{l,1} \Theta_l U_w^H W^2 A_{l-1}^H D_{l-1}^{-1}, \quad l = 2, \ldots, L. \]

(51)

The diagonal elements of \(\Theta_l\) are given by

\[ \theta_{l,i}^2 = \frac{1}{\lambda_{l,i}} \left( \frac{\sqrt{\lambda_{l,i}^2 + \nu_l \sigma_{l,i}^2}}{\nu_l \sigma_{l,i}} - \frac{1}{\sigma_{l,i}} \right)^+ \quad i = 1, \ldots, N_0 \]

(52)

where \(\sigma_{l,i}, i = 1, \ldots, N_0\), denotes the \(i\)th diagonal element of \(\Sigma_l\). The Lagrangian multiplier \(\nu_l > 0\) is determined by

\[ \sum_{i=1}^{N_0} \sigma_{l,i} (\frac{\sqrt{\lambda_{l,i}^2 + \nu_l \sigma_{l,i}^2}}{\nu_l \sigma_{l,i}} - \frac{1}{\sigma_{l,i}})^+ = P_l. \]

Obviously, \(R_l\) does not need to be calculated in the problem (48)-(49). Thus, the simplified relay design has a lower computational complexity than the algorithm which solves the problem (30)-(31). To apply the simplified relay design, we only need to change Step 3 in Table I to update \(F_l^{(n+1)}\) as (51) with fixed \(\{B_i^{(n+1)}\}\) and \(\{\nu_l^{(n)}\}\). It will be shown in the next section that for two-hop relay systems, the MI performance of this simplified relay design is slightly worse than that of the algorithm solving the problem (30)-(31).
IV. NUMERICAL EXAMPLES

In this section, we study the performance of the proposed source and relay precoding matrices design algorithms through numerical simulations. We simulate a flat Rayleigh fading environment where all channel matrices have entries with zero mean. To normalize the effect of the number of transmit antennas to the SNR, the variance of entries in $G_i$ is set to be $1/M_i$, $i = 1, \ldots, N_u$, and the variance of entries in $H_l$ is set to be $1/N_{i-1}$, $l = 2, \ldots, L$. All noises are complex circularly symmetric with zero mean and unit variance. In order to study the system MI versus the power constraint at the source and relay nodes, we assume that all relay nodes have the same transmission power constraint $P$, i.e., $p_l = P$, $l = 2, \ldots, L$, and each user’s transmission power budget is $Q$, i.e., $q_i = Q$, $i = 1, \ldots, N_u$. The variables $\varepsilon_1$ and $\varepsilon_2$ for stopping the iterations in the proposed algorithm are both set to be $10^{-3}$.

We compare the performance of the proposed algorithm described in Table I (denoted as Proposed Algorithm 1), the proposed algorithm with the simplified relay matrices design in (51) (denoted as Proposed Algorithm 2), and the naive amplify-and-forward (NAF) algorithm where all source and relay precoding matrices are scaled identity matrices satisfying the power constrains. In addition, for simulation examples with two-hop relay systems, we also compare with the Algorithm 6 proposed in [11].

In the first example, we simulate a two-hop ($L = 2$) MIMO relay system with $N_u = 2$, $M_1 = 3$, $M_2 = 2$, $N_1 = N_2 = 6$, and $P = Q = 20$dB. The MI from the proposed algorithms at different number of iterations ($\{B_i\}$, $\{F_i\}$, and $W$ are updated in each iteration) is shown in Fig. 2. It can be clearly seen that the MI from both algorithms increases with iterations and only a few iterations are required for both algorithms to converge. In fact, we observed in many simulations that the Proposed Algorithms 1 and 2 converge in $4 \sim 5$ and $10 \sim 12$ iterations, respectively. It can also be seen in Fig. 2 that although the Proposed Algorithm 2 saves per-iteration computational complexity by approximating $R_i$ as $W$, the Proposed Algorithm 1 has faster convergence speed and a better MI performance than the Proposed Algorithm 2.

The MI performance of all algorithms tested versus $P$ is shown in Fig. 3 with $Q = 20$dB. It can be seen that the system MI yielded by four algorithms increases as $P$ increases. Both proposed algorithms and the Algorithm 6 of [11] outperform the NAF algorithm in terms of the system MI. In this scenario, the proposed two algorithms yield almost the same MI as the Algorithm 6 of [11]. The MI performance of the Proposed Algorithm 1 is only slightly better than that of the Proposed Algorithm 2. Moreover, we observe from Fig. 3 that at high $P$ level (above 47dB), the increasing of the system MI versus $P$ is very marginal. The reason is that the performance of a MIMO relay system is subjected to both source power constraint $Q$ and relay power constraint $P$. When $Q$ is fixed, the performance of all algorithms has the saturation effect as $P$ increases.

The MI performance of all algorithms versus $Q$ is demonstrated in Fig. 4 with $P$ fixed at 20dB. Similar to Fig. 3,
all users have the same number of antennas with $M$. For the sake of notational simplicity, we assume that the proposed algorithms can be extended to multihop multiuser systems. The gap of two algorithms becomes smaller. This is because the condition of approximating $R_i$ as $W$ is in high SNR scenarios. As $P$ increases, $R_i$ is getting closer to $W$, and thus, the MI gap between two proposed algorithms becomes smaller.

Finally, Fig. 7 illustrates the MI performance of three algorithms versus $Q$ with $P$ fixed at 20dB. It is clear from Fig. 7 that both proposed algorithms have much higher MI than the NAF algorithm. Moreover, different from the two-hop system, the MI performance of Algorithm 1 is obviously better than that of the Algorithm 2 over the whole range of $Q$ in the four-hop system. Considering the convergence properties and the MI performance, Algorithm 1 is more suitable for multihop systems.

V. CONCLUSIONS

We have proposed source and relay precoding matrices design algorithms for a multiuser multihop MIMO relay system. By exploiting the link between the maximal MI and the WMMSE objectives, an iterative algorithm has been developed to maximize the system MI by solving the WMMSE problem at each iteration. It has been shown that the WMMSE matrix of the signal waveform estimation at the destination node can be decomposed into the sum of the WMMSE matrices at all relay nodes, which greatly reduces the computational complexity at a (moderately) high SNR environment. Numerical examples have shown the effectiveness of the proposed algorithms.
APPENDIX A

PROOF OF THEOREM 1

Using (13), the objective function (10) can be rewritten as
\[
\min_{\{B_i\}, \{F_i\}, W} \log |E_L|.
\]

The objective function (14) can be equivalently rewritten as
\[
\min_{\{B_i\}, \{F_i\}, W} tr(WE_L) - \log |W|.
\]

The derivative of (53) with respect to \(B_i\) or \(F_i\) is given by
\[
\frac{\partial \log |E_L|}{\partial M} = tr\left(\left(\frac{\partial \log |E_L|}{\partial E_L}\right)^T \frac{\partial E_L}{\partial M} \right) = tr\left(E_L^{-1} \frac{\partial E_L}{\partial M} \right)
\]

where \(M\) can be either \(B_i\) or \(F_i\) and the identity of \(\partial \log |X|/\partial X = (X^{-1})^T\) [24, (57)] is used.

Similarly, by using the chain rule of matrix derivatives and the identity of \(\partial tr(AX)/\partial X = A^T\) [24, (100)], we obtain the derivative of (54) with respect to \(B_i\) or \(F_i\) as
\[
\frac{\partial tr(WE_L) - \log |W|}{\partial W} = E_L^T - (W^{-1})^T
\]

By equating (57) to zero, we obtain (17). Thus with given \(\{B_i\}\) and \(\{F_i\}\), the weight matrix \(W\) minimizing (54) is given by (17).

APPENDIX B

PROOF OF THEOREM 2

The WMMSE matrix \(E_L\) can be rewritten as
\[
\hat{E}_L = W^{\frac{1}{2}} (I_{N_0} - A_{L-1}^H F_L^H H_L^H (H_L F_L D_{L-1} F_L^H H_L^H + I_{N_L})^{-1} H_L F_L A_{L-1}) W^{\frac{1}{2}}
\]

where \(A_{L-1} \triangleq A_{L-1} W^{\frac{1}{2}}\). The matrix inversion lemma (27) is used to obtain (58) and (60), and the identity of \(B^H (BCB^H + I)^{-1} B = C^{-1} - (CB^H BC + C)^{-1}\) is applied to get (59).

It can be seen that the first term in (60) is irrelevant to \(F_L\). Therefore, the problem of optimizing \(F_L\) can be written as
\[
\min_{F_L} tr(\hat{A}_{L-1}^H (D_{L-1} - F_L^H H_L^H H_L F_L D_{L-1} F_L^H H_L^H + D_{L-1})^{-1} \hat{A}_{L-1})
\]

s.t. \(tr(F_L D_{L-1} F_L^H) \leq p_L\).

By introducing \(\hat{F}_L = F_L D_{L-1}^\frac{1}{2}\), the problem (61)-(62) can be rewritten as
\[
\min_{F_L} tr(\Psi_{L-1}^H (\hat{F}_L^H H_L^H H_L F_L + I_{L-1})^{-1} \Psi_{L-1})
\]

s.t. \(tr(\hat{F}_L^H F_L^H) \leq p_L\)

where \(\Psi_{L-1} \triangleq D_{L-1}^{-\frac{1}{2}} A_{L-1} W^{\frac{1}{2}}\).

Let us introduce the EVD of \(H_L^H H_L = V_L A_L V_L^H\), and the singular value decomposition (SVD) of \(\Psi_{L-1} = U_L \Sigma_L V_L^H\), where \(A_L\) and \(V_L\) are \(N_L \times N_L\) matrices, the dimensions of \(U_L, \Sigma_L, V_L\) are \(N_L \times N_L, N_L \times N_L, N_L \times N_L\), respectively, and the diagonal elements of \(A_L\) and \(\Sigma_L\) are both sorted in descending order. Based on Lemma 2 in [15], the SVD of the optimal \(F_L\) is given by \(F_L = V_L L_1 \Sigma_L U_L^H\), where \(L_1\) is the \(N_0 \times N_0\) diagonal singular value matrix, and \(V_L L_1\) denotes the leftovermost \(N_0\) columns of \(V_L\). So we have
\[
\hat{F}_L = V_L L_1 \Sigma_L V_L^H \Psi_{L-1} V_L^H = T_L \Psi_{L-1}^H L_2
\]

where \(T_L \triangleq V_L L_1 \Sigma_L \Psi_{L-1} V_L^H\), and
\[
F_L = T_L W^{\frac{1}{2}} A_{L-1}^H D_{L-1}^{-1}
\]

Using (66) and the matrix inversion lemma (27), the second term in (60) can be rewritten as
\[
\hat{A}_{L-1}^H (\hat{A}_{L-1} - D_{L-1}^H H_L^H H_L T_L \hat{A}_{L-1} + D_{L-1})^{-1} \hat{A}_{L-1}
\]

\[
= \hat{A}_{L-1}^H [D_{L-1}^{-1} - D_{L-1}^{-1} \hat{A}_{L-1} \hat{A}_{L-1}^H D_{L-1}^{-1} \hat{A}_{L-1}^{-1}] D_{L-1}^{-1} \hat{A}_{L-1}^{-1}
\]

\[
= [T_L^H H_L^H H_L T_L + (\hat{A}_{L-1}^H D_{L-1}^{-1} \hat{A}_{L-1}^{-1})^{-1}]
\]

Substituting (67) back into (60) and using (23), we have
\[
\hat{E}_L = \hat{E}_{L-1} + (T_L^H H_L^H H_L T_L + R_{L-1})^{-1}
\]

where \(\hat{E}_{L-1} = W^{\frac{1}{2}} (I_{N_0} + A_{L-1}^H C_{L-1}^{-1} A_{L-1}) W^{\frac{1}{2}}\) is the WMMSE matrix at the \(L\)-th hop.

It can be seen from (68) that \(\hat{E}_L\) can be decomposed recursively. By replacing \(L\) with \(l\), we can get \(\Psi_{L-1}\) and \(T_L\) in a similar way as (58)-(68). It can be shown that the optimal \(F_l\) is given by \(F_l = T_l W^{\frac{1}{2}} A_{L-1}^H D_{L-1}^{-1}\), and \(\hat{E}_l\) is given by
\[
\hat{E}_l = \hat{E}_{l-1} + (T_l^H H_l^H H_l T_l + R_{l-1})^{-1}, l = 2, \ldots, L-1
\]

\[
\hat{E}_1 = W^{\frac{1}{2}} (I_{N_0} + F_l^H H_l^H H_l F_l)^{-1} W^{\frac{1}{2}}
\]

Combining (68)-(70), we obtain \(\hat{E}_L = W^{\frac{1}{2}} (I_{N_0} + F_l^H H_l^H H_l F_l)^{-1} W^{\frac{1}{2}} + \sum_{l=2}^{L} (T_l^H H_l^H H_l T_l + R_{l-1})^{-1}\).

Using (21), the transmission power consumed by each relay node in (8) can be rewritten as
\[
tr(F_l D_{L-1} F_l^H) = tr(T_l R_l T_l^H), \quad l = 2, \ldots, L
\]

ACKNOWLEDGMENT

The authors would like to thank the editor and anonymous reviewers for their valuable comments and suggestions that improved the quality of the paper.
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