ROBUST $H_\infty$ STABILISATION WITH DEFINITE ATTENUANCE OF AN UNCERTAIN IMPULSIVE SWITCHED SYSTEM

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Abstract

In this paper, we study the problem of robust $H_\infty$ stabilisation with definite attenuance for a class of impulsive switched systems with time-varying uncertainty. A norm-bounded uncertainty is assumed to appear in all the matrices of the state model. An LMI-based method for robust $H_\infty$ stabilisation with definite attenuance via a state feedback control law is developed. A simulation example is presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Robust $H_\infty$ stability and control problems of dynamic systems have attracted considerable attention for several decades. The main focus has been on $H_\infty$ problems for linear systems [10, 2], nonlinear systems [9], and systems without delay as well as with delays [5, 10, 2, 9] and so on. In recent years, interest has been extended to the robust $H_\infty$ stability problem of impulsive dynamic systems.

Some typical examples of impulsive systems can be found in [11]. One such system is used to model the population of a certain kind of insect via introducing its natural enemies at certain time instances. Another system models control of the reaction process of a chemical reactor by adding chemicals at certain time instances. In [7], it is pointed out that the dynamical behaviour of the total stock value of a particular investor can be described by an impulsive system. Some other examples can be found in [3] and [6]. In [3] and [4], various issues concerning the stability and robust stability of impulsive dynamic systems are studied using Lyapunov functions. In particular, the...
impulsive switched system with norm-bounded time-varying uncertainty has attracted much attention.

In this paper, we consider the robust $H_\infty$ control problem with definite attenuance of a class of uncertain impulsive switched systems. The problem is to design a feedback control law such that the closed loop impulsive system is asymptotically stable and robustly stable with definite attenuance and $H_\infty$ performance. Using the LMI approach, we derive a set of sufficient conditions for ensuring the existence of such a feedback control law. This feedback control law can then be obtained by solving a set of linear matrix inequalities.

The rest of the paper is organised as follows. In Section 2, a general class of uncertain impulsive switched systems with time-varying uncertainty is presented and some useful definitions are given. In Section 3, some sufficient conditions for robust $H_\infty$ stability with definite attenuance for the impulsive switched system are derived. A constructive method for constructing the corresponding feedback control law is then given. In Section 4, a numerical example is presented to illustrate how such a feedback control law is constructed. Finally, Section 5 concludes the paper.

2. Problem statement

Consider the following class of uncertain impulsive switched systems:

$$\begin{cases}
\dot{x}(t) = (A_{i_k} + \Delta A_{i_k}(t))x(t) + B_{i_k}e^{-\lambda t}w(t) + C_{i_k}u(t), & t \neq t_k, \\
\Delta x(t) = l_k(t, x) = D_kx(t), & t = t_k, \\
x(t) = 0, & t = t_0 = 0.
\end{cases} \tag{2.1}$$

Here $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $w \in \mathbb{R}^p$ is an uncertainty. Also $A_{i_k} \in \mathbb{R}^{n \times n}$, $B_{i_k} \in \mathbb{R}^{n \times p}$, $C_{i_k} \in \mathbb{R}^{n \times m}$ and $D_k \in \mathbb{R}^{n \times n}$ are constant real matrices, $\lambda$ is a positive constant, $k = 1, 2, \ldots, t_k \in \{1, 2, \ldots, s\}$ and $s$ is a positive integer. For each $i_k \in \{1, 2, \ldots, s\}$, $\Delta A_{i_k}(\cdot)$ is an unknown real norm-bounded matrix function representing the time-varying parameter uncertainty. An admissible uncertainty is assumed to be of the form

$$\Delta A_{i_k}(t) = F_{i_k} \Xi_{i_k}(t)H_{i_k}, \tag{2.2}$$

where $F_{i_k}$ and $H_{i_k}$ are known real constant matrices and $\Xi_{i_k}(t)$, $i_k \in \{1, 2, \ldots, s\}$, are unknown real time-varying matrices satisfying $\|\Xi_{i_k}(t)\| < 1$. Here $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$, $x(t_k^-) = \lim_{h \to 0^-} x(t_k + h)$, $t_k$ is an impulsive switched point, $k = 1, 2, \ldots$, and $t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ ($t_k \to \infty$ as $k \to \infty$).
Define the variable \( \xi(t) = e^{\lambda t}x(t) \) as the state variable of the impulsive switched system (2.1) and \( z(t) = E_{in}\xi(t) \), \( E_{in} \in \mathbb{R}^{q \times n} \), the control output. Let \( \bar{u}(t) = e^{\lambda t}u(t) \).

Then system (2.1) can be written as:

\[
\begin{align*}
\dot{\xi}(t) &= (\lambda I + A_{in} + \Delta A_{in}(t))\xi(t) + B_{in}w(t) + C_{in}\bar{u}(t), \\
\Delta \xi(t) &= I_{i}(t, \xi), \\
z(t) &= E_{in}\xi(t), \\
\xi(t) &= 0,
\end{align*}
\]

(2.3)

where \( \xi \in \mathbb{R}^{n} \) is the state, \( \bar{u} \in \mathbb{R}^{m} \) is the control input, \( w \in \mathbb{R}^{p} \) is the uncertainty and \( z \in \mathbb{R}^{q} \) is the control output. Other parameters are the same as those defined for system (2.1).

**Definition 1.** The uncertain impulsive switched system (2.1) is said to be robustly stable if the trivial solution \( x(t, t_0, x_0) = 0 \) of the functional differential equation (2.1) with \( u(t) = 0 \) is asymptotically stable with respect to all admissible uncertainties.

**Definition 2.** The uncertain impulsive switched system (2.1) is said to be robustly stabilisable if there exists a linear state feedback control law \( u(t) = \Gamma_{nk}x(t) \) with \( \Gamma_{nk} \in \mathbb{R}^{m \times n} \) such that the resulting closed loop system is robustly stable in the sense of Definition 1.

**Definition 3.** For given scalars \( \gamma > 0 \) and \( \lambda > 0 \), the uncertain impulsive switched system (2.1) is said to be robustly \( H_{\infty} \) stable with definite attenuance under any given switching law if the uncertain impulsive system has \( H_{\infty} \) performance, that is, \( \|z(t)\|_{\gamma} < \gamma w(t)\|_{\gamma} \) is satisfied, and the trivial solution \( \xi(t, t_0, x_0) = 0 \) of the functional differential equation (2.3) with \( u(t) = 0 \) is asymptotically stable with respect to all admissible uncertainties.

**Definition 4.** For given scalars \( \gamma > 0 \) and \( \lambda > 0 \), the uncertain impulsive switched system (2.1) is said to be robustly \( H_{\infty} \) stabilisable with definite attenuance if there exists a state feedback control law such that for any admissible uncertainty, the following conditions are satisfied under any given switching law:

(i) Robust stability: The resulting closed-loop system of the impulsive switched system (2.3) is asymptotically stable;

(ii) \( H_{\infty} \) performance: When a positive constant \( \gamma \) is given as the objective performance, \( \|z(t)\|_{\gamma} < \gamma w(t)\|_{\gamma} \).

In this paper, we shall derive a set of sufficient conditions to guarantee robust \( H_{\infty} \) stabilisation with definite attenuance for a class of uncertain impulsive switched systems. More specifically, for given scalars \( \gamma > 0 \) and \( \lambda > 0 \), our objective is
to obtain sufficient conditions to ensure robust stability with definite attenuance and, at the same time, achieve $H_\infty$ performance. Furthermore, they can be used to find a stabilising state feedback control law $u(t) = \Gamma_\infty x(t)$ such that the resulting closed loop impulsive system is robustly $H_\infty$ stable with definite attenuance. This state feedback control law can be obtained via solving a set of linear matrix inequalities.

3. Main results

First, we give several lemmas which are needed for the proofs of our main theorems.

**Lemma 3.1 ([8]).** Let $G \in R^{p \times q}$ be a matrix such that $G^T G \leq I$. Then

$$2x^T G y \leq x^T x + y^T y$$

for all $x \in R^p$ and $y \in R^q$. In the case $G = I$, (3.1) reduces to $2x^T y \leq x^T x + y^T y$.

**Lemma 3.2.** Let $P \in R^{n \times n}$ be a positive definite matrix and $Q \in R^{n \times n}$ a symmetric matrix. Then

$$\lambda_{\min}(P^{-1} Q)x(t)^T P x(t) \leq x(t)^T Q x(t) \leq \lambda_{\max}(P^{-1} Q)x(t)^T P x(t)$$

for all $x(t) \in R^n$.

**Lemma 3.3 ([5]).** Let $A$, $F$, $Z$ and $H$ be real matrices of appropriate dimensions with $\|Z\| \leq 1$. Then for any scalars $\varepsilon > 0$,

$$F Z H + H^T Z^T F^T \leq \varepsilon^{-1} F F^T + \varepsilon H^T H.$$

To proceed further, we assume that the following condition is satisfied:

**Assumption 3.4.** $\|w(t)\| \leq L \|\xi(t)\|$.

We now present our main results on robust $H_\infty$ stabilisation with definite attenuance.

**Theorem 3.5.** Let $\beta_1$ be the largest eigenvalue of the matrix

$$P_{\infty}^{-1}(I + D_k)^T P_k(I + D_k).$$

Suppose that Assumption 3.4 is satisfied. Then, for any given switching law, system (2.1) is robustly $H_\infty$ stable with definite attenuance if there exists a symmetric positive
definite matrix $P_i$, positive scalars $\gamma$, $\varepsilon$, $\lambda$ and $0 \leq \beta_k < 1$, $k \in N$, such that the following linear matrix inequalities are satisfied:

\[
\begin{bmatrix}
S(\lambda, P_i, \varepsilon) & \varepsilon^{-1/2} P_i F_i & P_i B_i & LI \\
\varepsilon^{-1/2} F_i^T P_i & -I & -I & 0 \\
B_i^T P_i & -I & -I & 0 \\
L & I & I & 0
\end{bmatrix} < 0 \quad \text{and} \quad (3.2)
\]

where

\[
S(\lambda, P_i, \varepsilon) = (\lambda I + A_i^T P_i + P_i (\lambda I + A_i) + \varepsilon H_i^T H_i).
\]  
(3.4)

**Proof.** For $t \in [t_k, t_{k+1}]$, define the following Lyapunov function for system (2.3) with $u(t) = 0$:

\[
V(t) = \xi^T(t) P_i \xi(t),
\]  
(3.5)

where $P_i$ is a positive definite symmetric matrix. Taking the time derivative of $V(t)$ along the solution of (2.3) with $u(t) = 0$, and then making use of (2.2), we obtain

\[
\dot{V}(t) = \xi^T(t) P_i \dot{\xi}(t) + \xi^T(t) P_i \dot{\xi}(t)
\]

\[
= \left[ \lambda \dot{\xi}(t) + \xi^T(t) (A_i + \Delta A_i)^T + w^T(t) B_i^T \right] P_i \xi(t)
\]

\[
+ \xi^T(t) [P_i (\dot{\xi}(t) + (A_i + \Delta A_i) \xi(t) + B_i w(t))]
\]

\[
= \xi^T(t) \left[ (\lambda I + A_i^T P_i + P_i (\lambda I + A_i) + H_i^T \Xi_i (P_i F_i)^T + P_i F_i \Xi_i H_i^T \right] \xi(t)
\]

\[
+ w^T(t) \left( \xi^T(t) P_i B_i^T \right) + \xi^T(t) P_i \dot{B}_i^T w(t). \quad \text{(3.6)}
\]

By Lemma 3.3, we have, for any $\varepsilon > 0$,

\[
H_i^T \Xi_i (P_i F_i)^T + P_i F_i \Xi_i H_i \leq (1/\varepsilon) P_i F_i F_i^T P_i + \varepsilon H_i^T H_i \quad \text{(3.7)}
\]

and, for any $\gamma > 0$,

\[
w^T(t) \left( \dot{\xi}^T(t) P_i B_i^T \right) + \dot{\xi}(t) \dot{w}(t)
\]

\[
\leq \gamma^{-1/2} \dot{\xi}(t) P_i B_i^T P_i \dot{\xi}(t) + \gamma \dot{w}^T(t) \dot{w}(t). \quad \text{(3.8)}
\]

Applying (3.7) and (3.8) with $\gamma = 1$ to (3.6), and using Assumption 3.4, it follows that

\[
\dot{V}(t) \leq \xi^T(t) \left[ (\lambda I + A_i^T P_i + P_i (\lambda I + A_i) + \dot{\varepsilon} P_i F_i F_i^T P_i + \varepsilon H_i^T H_i \right] \xi(t)
\]

\[
+ \xi^T(t) (P_i) \dot{B}_i^T (P_i) \dot{\xi}(t) + w^T(t) \dot{w}(t)
\]

\[
\leq \xi^T(t) S(\lambda, P_i, \varepsilon) + \varepsilon^{-1} P_i F_i F_i^T P_i + P_i B_i B_i^T P_i + L^2 \| \xi \|^2(t), \quad \text{(3.9)}
\]
where $S(\lambda, P, \epsilon)$ is defined by (3.4).

To ensure that
\[ \dot{V}(t) < 0 \quad \text{for} \quad t \in (t_k, t_k^+), \]
(3.10)
we choose $\epsilon > 0$ such that
\[ S(\lambda, P, \epsilon) + \epsilon^{-1} P_i F_i F_i^T P_i + P_i B_i B_i^T P_i + L^2 I < 0. \]
(3.11)

Let us now look at the function $V(t)$, defined by (3.5), at the switching points $t_k$. Then we have
\begin{align*}
V(t_k^+) &= \xi^T(t_k^+) P_i \xi(t_k^+) = \xi^T(t_k^-)(I + D_k)^T P_i (I + D_k)\xi(t_k^-) \\
&= \xi^T(t_k^-) P_{i-1}^{-1} (I + D_k)^T P_i (I + D_k)\xi(t_k^-) \\
&\leq \beta \dot{V}(t_k^-) < V(t_k^-),
\end{align*}
(3.12)
where $0 \leq \beta < 1$, is the maximum eigenvalue of $P_{i-1}^{-1} (I + D_k)^T P_i (I + D_k)$.

Combining (3.10) and (3.12), it follows that system (2.3) with $u = 0$ is asymptotically stable.

It remains to show that the uncertain impulsive system has $H_\infty$ performance. For this, it follows from (3.6)–(3.8) that
\begin{align*}
\dot{V}(t) &\leq \xi^T(t)[(\lambda I + A_i^T) P_i + P_i (\lambda I + A_i) + \epsilon^{-1} P_i F_i F_i^T P_i \\
&+ \epsilon H_i^T H_i] \xi(t) + \gamma^{-2} \xi^T(t) P_i B_i B_i^T P_i \xi(t) + \gamma^{-2} w^T(t) w(t) \\
&= \xi^T(t) S(\lambda, P, \epsilon) + \epsilon^{-1} P_i F_i F_i^T P_i + \gamma^{-2} P_i B_i B_i^T P_i + \epsilon E_i^T E_i \xi(t) \\
&- \xi^T(t) E_i^T E_i \xi(t) + \gamma^{-2} w^T(t) w(t).
\end{align*}

Now, by imposing
\[ S(\lambda, P, \epsilon) + \epsilon^{-1} P_i F_i F_i^T P_i + \gamma^{-2} P_i B_i B_i^T P_i + \epsilon E_i^T E_i < 0, \]
(3.13)
we obtain $\dot{V}(t) < -\|E_i\xi(t)\|^2 + \gamma^{-2}\|w(t)\|^2 = -\|z(t)\|^2 + \gamma^{-2}\|w(t)\|^2$ and hence
\[ \|z(t)\|^2 < -\dot{V}(t) + \gamma^{-2}\|w(t)\|^2. \]
(3.14)

Integrating both sides of (3.14), we have
\[ \int_0^\tau \|z(t)\|^2 dt < -\int_0^\tau \dot{V}(t) dt + \gamma^{-2} \int_0^\tau \|w(t)\|^2 dt, \quad \tau \in (t_k, t_{k+1}). \]
(3.15)
From (3.5) we see that $V(0) = 0$ and $V(t_k) > 0$. Since $0 \leq \beta_k < 1$, it follows that

$$
\int_0^\tau \dot{V}(t) \, dt = \int_0^{t_1} \dot{V}(t) \, dt + \int_{t_1}^{t_2} \dot{V}(t) \, dt + \cdots + \int_{t_{n-1}}^{t_k} \dot{V}(t) \, dt + \int_{t_k}^\tau \dot{V}(t) \, dt
$$

$$
= V(t_1) - V(0) + V(t_2) - V(t_1) + \cdots + V(t_k) - V(t_{k-1}) + V(\tau) - V(t_k)
$$

$$
\geq \sum_{i=1}^k (1 - \beta_i) V(t_i) + V(\tau) > 0.
$$

(3.16)

Combining (3.15) and (3.16), we have

$$
\|z(t)\|^2 = \int_0^\tau \|z(t)\|^2 \, dt < \gamma^2 \int_0^\tau \|w(t)\|^2 \, dt = \gamma^2 \|w(t)\|^2.
$$

Thus the $H_\infty$ performance is satisfied. Therefore system (2.1) is robustly $H_\infty$ stable with definite attenuation.

Finally, using the Schur complementary theorem [1], the inequalities (3.11) and (3.13) are equivalent to those of (3.2)–(3.3). This completes the proof.

In the case when there is no uncertainty in system (2.1) (or system (2.3)), that is, $\Delta A_{il} = 0$, we have the following result.

**Corollary 3.6.** Consider system (2.1) (or (2.3)) with $F_{ik} = 0$, $H_{ik} = 0$. Assume that Assumption 3.4 is satisfied. Let $\beta_k$ be the largest eigenvalue of $P_{ik}^{-1}(I + D_k)^T P_{ik} (I + D_k)$. Then the uncertain impulsive switched system (2.1) is robustly $H_\infty$ stable with definite attenuation if there exists a symmetric positive definite matrix $P_{ik}$, positive scalars $\gamma$, $\lambda$ and $0 \leq \beta_k < 1$, and $k \in N$ such that the following linear matrix inequalities are satisfied:

$$
\begin{bmatrix}
(\lambda I + A_{ik}^T) P_{ik} + P_{ik} (\lambda I + A_{ik}) & P_{ik} B_{ik} \\
B_{ik}^T P_{ik} & -I \\
I & -I
\end{bmatrix} < 0 \quad \text{and} \quad (3.17)
$$

$$
\begin{bmatrix}
(\lambda I + A_{ik}^T) P_{ik} + P_{ik} (\lambda I + A_{ik}) & \gamma^{-1} P_{ik} B_{ik} \\
B_{ik}^T P_{ik} & -I \\
\gamma^{-1} B_{ik}^T P_{ik} & -I
\end{bmatrix} < 0. \quad (3.18)
$$

**Theorem 3.7.** Let $\beta_k$ be the largest eigenvalue of the matrix $P_{ik}^{-1}(I + D_k)^T P_{ik} (I + D_k)$. For any given switching law, consider the uncertain impulsive switched system (2.1) and assume that Assumption 3.4 is satisfied. Then system (2.1) is robustly $H_\infty$ stable with definite attenuation if there exists a positive definite symmetric matrix
where the following linear matrix inequalities are satisfied:

\[
\begin{bmatrix}
S(\lambda, P_i, \varepsilon) & \varepsilon^{-1/2} P_i F_i \\
\varepsilon^{-1/2} F_i^T P_i & -I \\
P_i & 2C_i C_i^T - B_i B_i^T \\
L I & -I
\end{bmatrix} < 0 \quad \text{and} \quad (3.19)
\]

\[
\begin{bmatrix}
S(\lambda, P_i, \varepsilon) & \varepsilon^{-1/2} P_i F_i \\
\varepsilon^{-1/2} F_i^T P_i & -I \\
P_i & 2C_i C_i^T - \gamma^{-2} B_i B_i^T \\
E_i & -I
\end{bmatrix} < 0, \quad (3.20)
\]

where \(S(\lambda, P_i, \varepsilon) = (\lambda I + A_i^T) P_i + P_i (\lambda I + A_i) + \varepsilon H_i^T H_i\). Moreover, the feedback control law is given by \(u(t) = -C_i^T P_i x(t)\), which is a robust \(H_\infty\) stabilising controller.

**Proof.** For the given control law

\[
u(t) = \Gamma_i x(t), \quad (3.21)
\]

where \(\Gamma_i = -C_i^T P_i\). Then \(\tilde{u}_i(t) = \Gamma_i \tilde{x}(t)\). Define, for \(t \in (t_i, t_{i+1})\), the following Lyapunov function for (2.3): \(V(t) = \tilde{x}^T(t) P_i \tilde{x}(t)\), where \(P_i\) is a positive definite symmetric matrix. Taking the time derivative of \(V(t)\) along the solution of (2.3) with \(u(t)\) given by (3.21), we obtain

\[
\dot{V}(t) = \tilde{x}^T(t) P_i \tilde{x}(t) + \tilde{x}^T(t) P_i \tilde{\dot{x}}(t)
\]

\[
= [\lambda \tilde{x}^T(t) + \tilde{x}^T(t) (A_i + \Delta A_i) + w^T(t) B_i^T + \tilde{u}^T(t) C_i^T] P_i \tilde{x}(t)
\]

\[
= \tilde{x}^T(t) [(\lambda I + A_i^T) P_i + P_i (\lambda I + A_i) + H_i^T H_i] \tilde{x}(t) + W_i^T(t) [P_i B_i \xi(t) + \tilde{u}(t) \tilde{w}(t) + C_i \tilde{u}(t)]
\]

Then it follows that

\[
\dot{V}(t) \leq \tilde{x}^T(t) [(\lambda I + A_i^T) P_i + P_i (\lambda I + A_i) + \varepsilon^{-1} P_i F_i F_i^T P_i + \varepsilon H_i^T H_i] \tilde{x}(t)
\]

\[
+ \tilde{x}^T(t) (P_i B_i C_i^T P_i - 2P_i C_i C_i^T P_i) \tilde{w}(t) + w^T(t) \tilde{w}(t)
\]

\[
\leq \tilde{x}^T(t) [S(\lambda, P_i, \varepsilon) + \varepsilon^{-1} P_i F_i F_i^T P_i + P_i (B_i B_i^T - 2C_i C_i^T P_i + L^2 I)] \tilde{x}(t),
\]

where \(S(\lambda, P_i, \varepsilon)\) is defined by (3.4).

Now, using an argument similar to that given for (3.11), we choose \(\varepsilon > 0\) such that

\[
S(\lambda, P_i, \varepsilon) + \varepsilon^{-1} P_i F_i F_i^T P_i + P_i (B_i B_i^T - 2P_i C_i C_i^T P_i + L^2 I) < 0. \quad (3.22)
\]
Furthermore, using an argument similar to that given for (3.13), we impose that
\[
S(\lambda, P_n, \varepsilon) + \varepsilon^{-1} P_n F_n F_n^T P_n + \gamma^{-2} P_n B_n B_n^T P_n - 2 P_n C_n C_n^T P_n + E_n^T E_n < 0. \tag{3.23}
\]
Hence, using the Schur complementary theorem, the inequalities (3.22)–(3.23) can be written equivalently as (3.19)–(3.20).

4. A numerical example

Consider the uncertain impulsive switched system given by
\[
\begin{align*}
\dot{x}(t) &= (A_1 + \Delta A_1(t))x(t) + B_1 e^{-\lambda t} w(t) + C_1 u(t), \quad t \neq t_k, \\
\Delta x(t) &= I \Delta x(t), \quad t = t_k, \\
z(t) &= E_1 e^{\lambda t} x(t), \\
x(t) &= 0, \quad t = t_0 = 0, \\
\dot{x}(t) &= (A_2 + \Delta A_2(t))x(t) + B_2 e^{-\lambda t} w(t) + C_2 u(t), \quad t \neq t_k, \\
\Delta x(t) &= I \Delta x(t), \quad t = t_k, \\
z(t) &= E_2 e^{\lambda t} x(t), \\
x(t) &= 0, \quad t = t_0 = 0
\end{align*}
\tag{4.1}
\]
with
\[
A_1 = \begin{bmatrix} 0.8 & 0.5 \\ 0.8 & 0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.5 & 0.5 \\ 0 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.5 \\ 1 \\ 1.5 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_k = -0.5 \quad \text{and} \quad \Delta A_i(t) \quad \text{is the uncertainty matrix satisfying} \quad \|\Delta A_i(t)\| \leq 0.2.
\]
Then this system is in the form of system (2.1) (or (2.3)) with
\[
F_i = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix} \quad \text{and} \quad H_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]
Choose \( \gamma = \varepsilon = L = \lambda = 1. \) Then, by solving the corresponding versions of (3.19) and (3.20), we obtain, under any given switching law, positive definite symmetric matrices:
\[
P_1 = \begin{bmatrix} 2.6756 & 0.7351 \\ 0.7351 & 0.0941 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} 2.6887 & 0.2045 \\ 0.2045 & 0.0823 \end{bmatrix}
\]
such that the conditions of Theorem 3.7 are satisfied. Hence the required state feedback control law is: \( u(t) = \Gamma_i x(t), \) where
\[
\Gamma_1 = \begin{bmatrix} -3.0432 & -0.7822 \\ -2.0729 & -0.4617 \end{bmatrix} \quad \text{and} \quad \Gamma_2 = \begin{bmatrix} -2.8932 & -0.2868 \\ -2.9954 & -0.3979 \end{bmatrix}.
\]
FIGURE 1. Behaviour of the state $x(t)$ when $x_1(t) = x_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$.

FIGURE 2. Phase portrait of the uncertain impulsive switched system when $x_1(t) = x_2(t) = \sin(2\pi * 10t) * I$ and $w(t) = 0$. 
Figure 3. Behaviour of the state $\xi(t)$ when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi \times 10t) \ast I$ and $w(t) = 0$.

Figure 4. Behaviour of the state $x(t)$ with feedback control law $u(t) = \Gamma_x x(t)$ when $\Xi_1(t) = \Xi_2(t) = \sin(2\pi \times 10t) \ast I$ and $w(t) = 0$. 
FIGURE 5. Phase portrait of the uncertain impulsive switched system with feedback control law \( u(t) = \Gamma_0 x(t) \) when \( \Xi_1(t) = \Xi_2(t) = \sin(2\pi \ast 10t) \ast I \) and \( w(t) = 0 \).

FIGURE 6. Behaviour of the state \( \xi(t) \) with feedback control law \( u(t) = \Gamma_0 x(t) \) when \( \Xi_1(t) = \Xi_2(t) = \sin(2\pi \ast 10t) \ast I \) and \( w(t) = 0 \).
This state feedback control law asymptotically stabilises the impulsive switched system with norm-bounded time-varying uncertainty. Furthermore, it also guarantees robust stability with definite attenuance and has $H_\infty$ performance.

Assume that the switching law alternates the state of system (2.3) between (4.1) and (4.2). The numerical simulations are depicted in Figures 1–6. These figures show that the impulsive switched system with time-varying uncertainty is robustly stable. Furthermore, under the feedback control law obtained, the corresponding closed loop system is not only robustly stable with definite attenuance $\lambda = 1$ but also has $H_\infty$ performance.

5. Conclusion

We have derived a set of readily computable conditions in terms of linear matrix inequalities for a class of impulsive switched systems with time-varying uncertainty. Based on a positive definite solution of linear matrix inequalities, we can construct a robust $H_\infty$ state feedback control law, which guarantees robust stability with definite attenuance and gives rise to robust $H_\infty$ performance for the class of impulsive switched systems with norm-bounded time-varying uncertainty. An illustrative example was solved so as to demonstrate the effectiveness of the proposed approach.

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References