OPTIMAL CONTROL COMPUTATION FOR DISCRETE TIME
TIME-DELAYED OPTIMAL CONTROL PROBLEM WITH
ALL-TIME-STEP INEQUALITY CONSTRAINTS

BIN LI¹², KOK LAY TEO¹² AND GUANG REN DUAN¹²

¹Center for Control Theory and Guidance Technology
Harbin Institute of Technology
92 Xi Da Zhi Street, Harbin, 150001, P. R. China
bin.li@postgrad.curtin.edu.au
k.l.teo@curtin.edu.au
g.r.duan@hit.edu.cn

²Department of Mathematics and Statistics
Curtin University of Technology
GPO Box U1987, Perth, WA, 6845, Australia

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ABSTRACT. In this paper, we consider a class of discrete time optimal control problems
with time delay and subject to nonlinear all-time-step inequality constraints on both the
state and control. By using a constraint transcription technique in conjunction with
a local smoothing method, the problem is approximated by a sequence of discrete time
optimal control problems with time delay and subject to nonlinear inequality constraints in
canonical form. Rigorous analysis is carried out, showing the convergence of the optimal
solutions of the approximate problems to the optimal solution of the original problem.
We then move on to consider a general class of discrete time optimal control problem
with time delay and subject to nonlinear constraints in canonical form. A computational
method is developed based on the sequential quadratic programming (SQP) approximation
scheme with active set strategy. It solves the discrete time optimal control problem with
time delay and subject to canonical constraints as a nonlinear optimization problem.
As an application, we consider a tactical logistic decision analysis problem, which is
formulated as a discrete time optimal control problems with time delay and subject to all-
time-step inequality constraints. Using the computational method proposed, this practical
problem is solved effectively, producing much better results than those obtained in existing
literature.

Keywords: Time delayed system, Discrete time system, Optimal control, All-time-step
inequality constraints, Constraint transcription, Tactical logistic

1. Introduction. For many natural and man-made systems, inherent delays exist during
the transmission of information between different parts of the systems. As a consequence,
it gives rise to time delayed systems for which the evolution of current states depends
on the past and present values of states and controls. Optimal control of time delayed
systems has been an active research area since 1960s. For problems involving continu-
ous time systems with time delay, many papers are now available. See, for example,
[1-16]. Amongst these references, several computational methods (see [3-6,8-19]) are sug-
gested. For problems involving discrete time systems with time delay, there are much
less papers available in the literature. In [20], Kuhn-Tucker theorem of nonlinear pro-
gramming (see [22]) is used to derive a discrete maximum principle similar to Pontryagin
maximum principle for an optimal discrete time system with a pure delay. However, no
efficient computational algorithm is proposed using this discrete maximum principle. In