An Estimator to Reduce Mail Survey Nonresponse Bias in Estimates of Recreational Catch: a case study using data from the *Panulirus cygnus* fishery of Western Australia

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This is presented for the Degree of Doctor of Philosophy of Curtin University

August 2013
DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made.

This thesis contains no material that has been accepted for the award of any other degree or diploma in any university.

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ABSTRACT

Recreational fishing is partaken by approximately 11.5% of the world’s population, which retains an estimated global catch of 47 billion fish (and invertebrates) annually. In early 2000 an estimated 19.5% of Australians fished at least once in the prior 12 month period and retained an estimated total catch of 72 million fish. Studies suggest that this level of fishing has an impact on fish populations and thus, there is an increasing need for accurate estimates of the recreational catch and effort to be used in any formal assessment concerning the health of these stocks. Mail recall surveys are widely used to gather data for making these estimates, due to its ease of implementation and relatively low cost compared to other survey methods, particularly for a diffuse fishery that operates over a large spatial scale. Typically, a significant number of mail questionnaires (i.e. survey form) are not returned, which raises concerns of nonresponse bias (particularly if fishers are more likely to respond compared with non-fishers). If this is the case, then applying the ratio estimator ($\hat{\gamma}_0$) using returned questionnaires to estimate the mean of a dichotomous variable, such as the proportion of the surveyed population that fished in season $t$ ($\hat{p}_t$), required to estimate the total number of fishers, will be biased. This research aims to identify alternative estimators to $\hat{\gamma}_0$ that are less affected by nonresponse bias.

Weighting methods are widely used to adjust for survey nonresponse bias. These methods rely on identifying all auxiliary data (AD) that is correlated with the nonresponse rate and the response variable being measured and then, being able to obtain these data for both respondents and nonrespondents. Calibration is also used to identify a correction factor to standardize the estimate from a mail survey (MS) to one from a more accurate, but generally more expensive, survey method. This requires running the more reliable survey method, such as a phone-diary survey (PDS), in parallel to the MS for multiple seasons. Once a correction factor is calculated, the running of both surveys concurrently is required intermittently to assess continued validity in the correction factor as bias in the MS may change over time. Additionally the MS occurs some time after the fishing activity has occurred, therefore the correction factor also adjusts for any recall bias. The alternate estimators to $\hat{\gamma}_0$ that were identified in this research avoid the requirement of having
to identify the AD for weighting methods and attempts to explain the magnitude (or component) of the correction factor that accounts for nonresponse bias without the added expense of running the more reliable, but usually much more expensive, survey.

Western rock lobsters (*Panulirus cygnus*) are found along the West Australian coast from Augusta to Coral Bay. This population supports the largest single species commercial fishery in Australia and an important recreational fishery. In the last decade, seasonal commercial catches have ranged from 5,500 to 14,000 t, worth approximately A$200 – 400 million per annum, and up to 47,000 people in any one season are licensed to fish recreationally for this species by potting or diving. An end of season MS has been used to estimate the total catch and effort of this fishery since 1986/87. A more expensive, but more reliable, PDS was conducted for 7 of these seasons.

The sampling frame of these surveys was the western rock lobster database; a list of recreational fishers that purchased a licence, regardless of whether they eventually fished or not. This thesis identifies key differences between the collected responses from the MS and PDS for the recreational western rock lobster fishery. These comparisons indicated that a person was more likely to return the survey form if they actually fished and that ‘avid’ fishers (i.e. those that fished more often) were more likely to return the survey form than those that fished less often. These observations explain why the total catch estimates from applying $\hat{\gamma}_o$ to the returned MS data are approximately twice that of the PDS and why a correction factor that standardizes the raw MS total catch estimates to the more reliable PDS is desirable.

Motivated by the observation that people who fished were more likely to return the survey form, a result that was evidenced in other studies, alternatives to $\hat{\gamma}_o$ for estimating the mean of a dichotomous variable $\gamma$ (such as $\hat{p}_t$) which is less affected by this type of bias were identified. Multinomial ($\hat{\gamma}_m$) and expectation ($\hat{\gamma}_e$) estimators were proposed and using root-mean-square-error (RMSE) as the criterion, simulations demonstrate that on average, the expectation estimator ($\hat{\gamma}_e$) produces less biased estimates than $\hat{\gamma}_o$ for all $\gamma$. If it can be assumed $\gamma \leq 0.65$, then the multinomial estimator ($\hat{\gamma}_m$) produces even more accurate estimates than $\hat{\gamma}_e$ but at the
risk of being more biased than even $\hat{\gamma}_o$ when this is not true. When $\gamma \leq 0.65$ the bias in $\hat{\gamma}_o$ ranges between 25% and 131% and using $\hat{\gamma}_m$ and $\hat{\gamma}_r$ reduces this bias by 53 – 100% and 48 – 61%, respectively. Approximate variance functions for each estimator were derived, and whilst they tend to marginally underestimate the true variances, they lead to 95% Clopper-Pearson confidence intervals (generally suspected of being too conservative) that closely approximate the desired level of confidence.

The time series of western rock lobster MS data was used to demonstrate how to choose between $\hat{\gamma}_r$ and $\hat{\gamma}_m$, for a real data set where $\gamma$ is unknown whilst avoiding the risk of actually further biasing estimates. To maximize the opportunity of reducing the affects of nonresponse bias, the total catch equation for the recreational rock lobster fishery was reformulated to not only include $\hat{\gamma}_n$ but also the proportion of those fishers who were avid ($\hat{\gamma}_v$). A comparison of the resulting estimates of number of fishers ($\hat{L}_t$ - a function of $\hat{\gamma}_n$) and catch per fisher ($\hat{F}_t$ - a function of $\hat{\gamma}_n$), to those estimated using $\hat{\gamma}_o$, was made. Estimates of $\hat{L}_t$ using $\hat{\gamma}_m$ from the MS were more similar to those of the PDS than when using $\hat{\gamma}_o$. The same observation was also made for $\hat{F}_t$, although not with the same measure of closeness as obtained for $\hat{L}_t$. The difference in success for $\hat{L}_t$ and $\hat{F}_t$ may be partly due to a recall bias in the MS where the inaccurate reporting of days fished may result in fishers being misallocated to an avidity class where as a person was more likely to accurately recall whether or not they did go fishing.

To implement effective management changes to control the total catch take, managers are aided by understanding the drivers of catch, therefore models of $\hat{L}_t$ and $\hat{F}_t$ were identified for each fishing method (potting and diving). With exception to a scaling factor for $\hat{F}_t$, it was seen that within fishing method, the optimal models were very similar for having used $\hat{\gamma}_o$, $\hat{\gamma}_m$ to estimate $\hat{\gamma}_n$ and $\hat{\gamma}_v$. The inclusion of a measure of lobster abundance improved the models of $\hat{F}_t$. Interestingly however, lobster abundance only improved the model of $\hat{L}_t$ for potting, not diving. This may suggest that fishers are more likely to put pots in the water when lobsters are more abundant and easier to catch but, presumably due to the leisure activity of diving in itself, are still as likely to go diving even in years when lobsters are less abundant.
Given that potting accounts for a significant share of the recreational catch, this indicates that the proportion of total catch (recreational + commercial) taken by the recreational sector is likely to be smaller in lower catch years. This suggests that if managers want to maintain a fixed proportion of share allocation between the two sectors each season then employing a conservative management policy in higher abundance years (e.g. lower bag limits and restricted effort), but with a slackening of these restrictions in leaner years, may be necessary.
ACKNOWLEDGEMENTS

Firstly, I thank Drs Roy Melville-Smith and Henry Cheng who introduced me to the recreational western rock lobster mail survey data all those years ago and gave me the opportunity and advice since then, that has continued to motivate and result in the completion of this research. Thanks to my supervisor Prof. Lou Caccetta for his guidance that was required for presenting this research.

My thanks also go to Department of Fisheries (DoF) for providing me with the opportunity to undertake this research. Thanks also to its many staff for their constructive criticism of various draft chapters of this thesis. In particular I thank Dr Norm Hall for the time he gave me discussing general ideas that led to much improvement in this research. I also thank the “library staff” for sourcing some of those difficult to find reference materials cited throughout.

My appreciation also go to the many people who took the time to fill in the mail survey forms or participate in the phone-diary surveys that formed an important basis of this research; as well as the many DoF staff over the years who have been involved in the running of these surveys.

Last but certainly not least, I thank my mum for her encouragement and support over the years in both my education and life in general.
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Chapter 1

Introduction

Recreational fishing is partaken by approximately 11.5% of the world’s population that retains an estimated global catch of 47 billion fish annually (Cooke & Cowx 2004). Henry & Lyle (2003) conducted a survey in Australia that estimated 19.5% of its population undertook recreational fishing at least once in a 12 month period and retain an estimated total catch of 72 million finfish. Post et al. (2002) and Coleman et al. (2004) have completed studies that suggest that this level of fishing is having an impact on fish populations and thus, there is an increasing need to include recreational catches, and not just commercial, when assessing the health of these fish stocks (Ferrel & Sumpton 1998; Leigh & O’Neill 2003; Lyle et al. 2000; Mitchell et al. 2008; FAO 2011). These assessments may lead to management changes that better control the various fishing pressures on these stocks to ensure the sustainability of the fish species into the future.

Knowing the number of people who fished (fishers) in a given season is fundamental to establish accurate total catch estimates for that fishery. Historically, most commercial fisheries are monitored through mandatory reporting of catch and effort but this is rarely the case for recreational fisheries and hence, survey methods are employed to estimate the number of fishers and their average catch (Jones & Pollock 2013).

This research is primarily concerned with improving the estimate of total recreational catch using data from a mail recall survey method (mail survey - MS) by better estimating the number of fishers. Methods currently exist for doing this but they generally require identifying and gathering various ‘important’ auxiliary data or solving complex models. This research identifies estimators that can be directly and simply applied to the returned MS data without the additional overheads required by these alternative methods.
Participants in recreational fisheries can range from infrequent fishers who may only fish once a year to those who fish regularly across the season. Different fisheries may also have numerous access points for these fishers such as boats launched from harbors, marinas and private docks; or from the shoreline on piers, jetties and beaches. Given such differences the survey design used to estimate the total catch and effort that is representative of the entire fishing population may vary from fishery to fishery. This design is also dependent on the specific objectives required to inform the management questions as well as the temporal and spatial scales to be covered, the funds available to conduct the survey and whether or not the likely fishers are readily identifiable from a reference frame such as a fisher licence database. In choosing which survey design to employ there is often a trade-off between costs and biases (Thomson 1991).

There are two broad categories of survey methods used to estimate recreational catch and effort: onsite surveys that interview fishers during or immediately at the completion of fishing; and offsite surveys that collect information from fishers some time (even months) after they have completed their fishing activity (Hartill et al. 2011). These survey types have specific strengths and weaknesses. Rather than use just one survey method, mixed-mode data collection methods have been used where various survey types are run and used to estimate those parameters for which it is deemed more reliable (Pollock et al. 1997; Peytchev 2013), given the dynamics of the particular fishery under study.

In this Chapter a brief overview of the advantages and disadvantages of the various offsite and onsite survey methods commonly used to estimate total recreational catch is presented. This is followed by a brief description of the survey methods used over time in the western rock lobster (*Panulirus cygnus*) recreational fishery in Western Australia. This is a fishery with a long time series of catch estimates calculated using a MS method and for some seasons, estimates from a generally accepted more accurate phone-diary survey (PDS). These data sets will provide a case study for the research developed throughout. Finally, the main objectives of this research are outlined.
1.1 Offsite Methods

Offsite survey methods contact fishers away from the fishing grounds some time after they have completed their fishing activity (e.g. Connelly et al. 2000; Sparrevohn & Storr-Paulsen 2012). These fishers are randomly sampled from a fisher licence frame or some other list such as a telephone book. Often used offsite survey methods of recreational fishers include: mail, telephone, diary and phone-diary surveys.

The MS is often used due to its low cost compared to other survey sampling methods (Fox et al. 1998). Typically, at the end of the time period under study a survey form is sent to a random selection of people who were able to fish during that time. It is unusual for all the sent survey forms to be returned and this raises suspicion of a nonresponse bias where respondents differ to nonrespondents in terms of their would-be responses to the questions in the survey (Brown 1991; Strayer et al. 1993; Stein et al. 1999). Surveying people on activity that has already occurred as much as many months earlier, MSs are also suspected of suffering from a recall bias (Connelly & Brown 2011). Further to these biases, a self-administered survey such as this, where an independent person does not verify the catches, may also suffer from other measurement biases such as species misidentification.

Also interviewing a random sample of the referred population at the end of the period under study, telephone surveys suffer from the same biases as the MS. With the personal contact of an interviewer asking the questions however, these biases are expected to be reduced due to the usually higher response rates compared to the MS and reduced ambiguity in the survey questions with the survey respondent being able to seek clarification from the interviewer and vice-versa (Lavrakas 1993). The cost of the interviewer’s time however, can make this method significantly more expensive than the MS.

To reduce the likelihood of recall bias, diary surveys have been used. Rather than at the end of the study period, this survey method enrolls a random sample of people before the start of the fishing period to participate in the survey. These people are then given a diary to record their future fishing activity. At the end of the survey
period, the diaries are then returned to the collection agency. This survey method benefits by allowing people to record their fishing activity as it happens as opposed to recalling it some time later. This method can still be affected by a nonresponse bias however, where over time people lose interest in filling the diary and results in to incomplete information.

The PDS (McGlennon 1999; Lyle et al. 2002; Ryan et al. 2009) is generally considered to be less affected by nonresponse and recall bias than mail, telephone and diary surveys. This method is similar to the standard diary survey except that interviewers make regular contact with diarists during the fishing season to collect their fishing information. The rapport developed between participant and interviewer during the season aims to improve the quality of data collected and leads to reduced nonresponse. As with telephone surveys however, the time required of interviewers adds significantly to the overall cost of this survey type over the other offsite methods.

1.2 Onsite Survey Methods

Onsite survey method designs interview anglers in the fishing grounds. Experienced interviewers interact with fishers while they are fishing or moving from the fishing grounds from various access points (Hindsley et al. 2011). Unlike offsite methods, which rely on the fisher’s memory, onsite methods do not rely on fisher self-reported data but instead, the inspection of the catch by trained interviewers which leads to reduced measurement bias.

Instead of drawing a random sample from a sampling frame of people able to fish, onsite methods randomly sample from a period of time-of-day and locations. This schedule of randomly drawn time-of-day and locations are then visited by interviewers who complete a series of interviews of people in the fishing ground or access point. This collected data is then scaled up to the population using various methods (Jones & Pollock 2013).
The survey design used is dependent on the number of access points and available interviewer resources.

When a limited number of access points exist then the ‘access point’ survey is usually appropriate (Imber et al. 1991). This requires interviewers remaining at a particular access point and interviewing fishers as they enter and leave through this point e.g. as boats are launched and retrieved at a boat ramp.

When there are a limited number of access points but still too many for available resources to adequately construct a schedule of ramp visits that covers all time and location levels, ‘bus route’ surveys are used (Kinloch et al. 1997). Generally, a bus route design requires grouping access points that can all be realistically visited by the same interviewer within a particular time frame. When a group of ramps is scheduled, each ramp is visited one at a time by the same interviewer. The interviewer remains at each access point for a specified period of time before moving to the next ramp. The ramps are visited in order of the chosen direction with the first ramp being visited usually being randomly selected.

When the fishing ground has many or an unlimited number of access points (e.g. a long stretch of beach where people access from many different carparks), a ‘roving’ survey is preferred (McCormick et al. 2012). On the water, interviewers roam in a boat to make contact with fishers. On shore, interviewers travel by foot or bicycle looking for fishers to interview.

Onsite method designs are reliable for getting detailed information at a catch per fisher per day level (Imber et al. 1991) since the catch of each fisher can be verified by trained interviewers ensuring accurate species identification and length measurements, and consistency of the catch and effort data that are collected. The number of access points over a large spatial scale however, can make the cost of visiting all access points prohibitive and can lead to a decision to only survey major fishing locations, which can lead to a non-coverage bias i.e. each fisher not equally likely to be surveyed. These methods also suffer from a ‘length of stay bias’ – the probability that a particular fisher will be intercepted is proportionally related to the length of time they spend fishing (Jones et al. 1990); and an avidity bias - the
disproportionate representation of ‘avid’ anglers (Thomson 1991; Shonkwiler & Englin 2009). This means that effort per fisher will usually be overestimated given that a fisher that fishes for longer and/or more often is more likely to be interviewed.

For a given number of interviews, the costs of onsite survey methods tend to be much higher than for offsite methods. These methods are usually more cost-effective however, at providing estimates at a higher resolution than offsite surveys when the fishery operates over small spatial scales (Hartill et al. 2011). When many access points to the fishery exist NRC (2006) suggested that conducting onsite surveys for such a fishery has “physical, financial, and operational constraints that often lead to spatial or temporal biases in onsite sampling coverage that are not adequately accounted for in the estimation procedure”.

To convert the catch rate information from completed interviews, an estimate of the total number of fishers is required and is typically estimated by making a head count of the people in the fishing ground at various time intervals. During busy times, this can require employing additional field staff to make these counts so that interviews can continue. For boat based fishing, video camera surveillance of boat launch and retrievals at ramps, vehicle counter systems to count vehicles using the retrieval lane, camera snapshot counts of trailers and counts of tickets in car parks associated with boat ramps, have been used to improve the precision of these counts. Aerial surveys have also been used, particularly for counting the number of shore based fishers.

Due to safety concerns for interviewers working at night, onsite surveys may be restricted to reporting on daytime fishing. Darkness can also prohibit accurate counting of the number of fishers in the fishing ground at that time due to the lack of light. These issues do not exist for offsite survey methods.

1.3 Mixed-Mode Data Collection Survey Methods

With the different survey methods having their own strengths (summarized in Table 1.1), some survey designs use two or more of these surveys in combination to get the
### Table 1.1: A comparison of different designs that are commonly used to survey anglers. (Modified and updated from Imber et al. 1991 and Hartill et al. 2011).

<table>
<thead>
<tr>
<th>Survey design</th>
<th>Application</th>
<th>+ Advantages</th>
<th>- Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access Intercept</td>
<td>Useful for a fishery with a small number of access sites.</td>
<td>+ Generates accurate estimates of catch and effort.</td>
<td>- (A) Only visit one or two sites per day; (B) requires a priori knowledge of the utilization of access sites; (C) reducing measurement error of total effort can add significantly to costs; and (D) safety concerns for interviewers restricts method to daylight hours.</td>
</tr>
<tr>
<td>Bus route</td>
<td>Designed for use in large spatial areas with limited access points.</td>
<td>+ Requires a small number of personnel to perform survey.</td>
<td>- (B); (C); (D); and designing routes is complex.</td>
</tr>
<tr>
<td>Roving creel</td>
<td>Designed for use in large spatial areas with unlimited access points.</td>
<td>+ Requires a small number of personnel to perform survey.</td>
<td>-(B); (C); (D); effort is overestimated due to 'length of stay' bias; and information is collected for incomplete fishing trips.</td>
</tr>
<tr>
<td>Aerial</td>
<td>Estimate effort in a large spatial area.</td>
<td>+ Generates accurate number of boats and shore fishers; and can identify angler spatial and temporal trends.</td>
<td>- Difficult to differentiate between commercial and recreational boats; and inclement weather may hamper accuracy.</td>
</tr>
<tr>
<td>Mail</td>
<td>Useful in diffuse access fisheries</td>
<td>+ (E) Inexpensive; and (F) far more contacts with fishers can be made for a reasonable cost, compared to onsite members.</td>
<td>- Can have high nonresponse rate which can lead to a bias; and anglers suffer from limited recall of previous fishing activity as time since that fishing activity grows.</td>
</tr>
<tr>
<td>Telephone</td>
<td>Useful in diffuse access fisheries.</td>
<td>+ (E), (F); reduced nonresponse as compared to the mail survey; and captures fishing activity at all types of sites.</td>
<td>- Fisherman exhibit recall problems about species caught, number of fish caught, and time spent fishing.</td>
</tr>
<tr>
<td>Diary</td>
<td>Useful in diffuse access fisheries.</td>
<td>+ Inexpensive; can identify angler spatial and temporal trends; and minimizes recall bias.</td>
<td>- Can suffer from nonresponse bias.</td>
</tr>
<tr>
<td>Phone-diary</td>
<td>Useful in diffuse access fisheries.</td>
<td>+ Diary adds as prompt for fisher to record fishing activities to reduce reliance on fishers recall; and regular contact creates rapport between diarist and interviewer which reduces nonresponse.</td>
<td>- Cost of interviewer makes method more expensive than say, a mail survey.</td>
</tr>
</tbody>
</table>
best out of each. In a fishery covering a vast area that has many access points for example, an access intercept survey could be used for a select few access points to estimate the probability that a person in the fishing ground was fishing and their average catch rates. An aerial survey could then be used to estimate the number of people in the fishing ground so the data collected from the access intercept survey can be used to estimate total catch. In a fishery where people are required to be licensed, replacing the aerial survey with a telephone or MS could reduce costs further although at the expense of additional biases.

Only imagination and available resources limit the ways in which different survey methods can be used together to make required estimates. The survey designer needs to be mindful however, that different modes of survey may have their own measurement errors (de Leeuw 2005). For example, the way a person may respond to the same question from a self-administered survey to one where prompted by an interviewer, may differ (Turner et al. 1992).

1.4 Survey Methods Used in the Western Rock Lobster Recreational Fishery

The western rock lobster (*Panulirus cygnus*) recreational fishery is an example of a diffuse access fishery that has a licence database to provide names and contact details of possible fishers for which the offsite survey methods such as the MS and PDS (Table 1.1) are suitable for estimating total catch and effort.

In 1986 a recreational licence specific to rock lobster fishing was introduced that allowed people to fish and retain western rock lobsters for personal consumption. Western rock lobster are distributed along the West Australian coast from latitude 21°44’S to 34°24’S in water as shallow as 1m and as deep as 200m. The remoteness of some locations however, has led to recreational fishing mainly occurring off the mainland coast from Augusta to Kalbarri (Figure 1.1). The two fishing methods primarily used by recreational fishers are potting and diving which are typically completed from boats of a size that tend to restrict their fishing to well-protected
waters in depths less than 40 m. There are up to 250 boat ramps and ca. 800km of coastline between Augusta and Kalbarri from which fishers can launch their boats.

Figure 1.1: Western rock lobsters are found along the West Australian coast between latitudes 21°44′S and 34°24′S and can be found in water as shallow as 1m and as deep as 200 m (blue region). Due to remoteness of location, recreational fishing mainly occurs between Kalbarri and Augusta.
Not all people who are licensed to do so, fish for rock lobster. Since the introduction of the rock lobster specific licence for the start of the 1986/87 season, a MS method has been used at the end of each 7.5 month long season (currently 15 November – 30 June) to estimate the total catch and effort of this fishery (Melville-Smith & Anderton 2000).

In summary, the western rock lobster MS is initiated by mailing a questionnaire (i.e. survey form) to between 4000 and 8000 people, randomly selected from those who were licensed to fish for rock lobster, at the end of a season \((t)\). The returned survey forms are then summarized into the number reporting having fished \((n_{t,f})\), not fished \((n_{t,\bar{f}})\) and the average catch retained by each person that reported having fished \((\hat{F}_f)\). These variables are used to estimate the proportion of the licensed population that fished \((p_t)\), the number of these licensees \((N_t)\) that fished \((L_t = N_t p_t)\), and the total catch \((T_t = L_t F_t)\).

Using data from a MS, the ratio estimator \(\hat{\gamma}_o\) estimates the proportion of a surveyed population that has some characteristic \(x\) (e.g. fished) as the proportion of survey returns that reported having \(x\) \((n_x)\) compared to not \((n_{\bar{x}})\) i.e. \(\hat{\gamma}_o = n_x / (n_{\bar{x}} + n_x)\). Traditionally \(\hat{\gamma}_o\) has been used to estimate \(p_t\) from the summary of returned survey forms for the western rock lobster recreational fishery i.e. \(\hat{p}_t = n_{t,f} / (n_{t,\bar{f}} + n_{t,f})\).

For the western rock lobster MS the return rate of the survey form has ranged between 40% and 60% each season (Melville-Smith & Anderton 2000). The \(\hat{\gamma}_o\) results in unbiased estimates of \(p_t\), and hence \(L_t\), when people who did and did not fish are equally likely to return the survey form (see Chapter 2).

The western rock lobster fishery has recently undergone a formal process to allocate catch shares to the recreational and commercial sectors (Crowe et al. 2013) which has led to the proportion of total sustainable catch being set at 5% and 95% respectively for each sector. This Integrated Fisheries Management (IFM) process of formal allocation includes an expectation that the catch for each sector will be known.
so that management adjustments can be made to regulate catch, if required, to maintain these proportions.

The accuracy of the MS to estimate total western rock lobster recreational catch is dependent on such things as whether or not people who did and did not fish are equally likely to return the survey form, and a persons ability to accurately recall their total retained catch from fishing that may have occurred as long as 7.5 months earlier. To determine if MS estimates of total catch could be used by managers to reliably assess the total catch of this fishery, it was decided to also run a much more expensive, but much considered more reliable, PDS method for several seasons.

The PDS design as described by Baharathah (2007), which is similar to that used by Fishcount in the Northern Territory (Coleman, 1998; Lyle et al., 2002), has been carried out in seven seasons (2000/01, 2001/02, 2004/05 – 2008/09). For each season this method required running a screening phase of the currently licensed people ca. 6 weeks prior to the start of the season to enrol a quota of willing diarists, which varied between 400 and 800 people each season. The successfully screened diarists were then sent a diary in which they could record their daily rock lobster fishing activity for the upcoming season (when, where, method of fishing and how many lobsters they retained and released). Diarists were phoned each month and an interviewer recorded the details of their fishing activity for that period. The same interviewer was maintained for each diarist throughout the survey to develop rapport between diarist and interviewer to encourage continued participation and reporting of accurate data. Similar to the MS, the data collected from the PDS was then used to estimate total catch and effort.

A comparison of a number of seasons showed that the MS total catch estimates were approximately twice that from the corresponding PDS (IFAAC 2007; Hartill et al. 2011) and indicates that nonresponse and/or recall bias may be having a significant impact on the annual MS of western rock lobster recreational fishers.
1.5 Summary and Overview

Compared to using the commonly used ‘ratio’ estimator ($\hat{\gamma}_{(0)}$), this research is focussed on identifying an estimator that reduces the impact of nonresponse bias when estimating the proportion of a surveyed population who fished, to improve total catch estimates calculated using data collected from a MS.

The objectives of this research are:

i) Identify the impact of nonresponse on the reported number of fishers for a MS of people who were licensed to fish for western rock lobster;

ii) Using the identified directions of bias in (i), identify an estimator that is less affected by nonresponse bias than $\hat{\gamma}_{(0)}$ when estimating the proportion of people in the population who fished; and

iii) Demonstrate the application and usefulness of the proposed estimator in (ii) at removing nonresponse bias from the recreational western rock lobster fisheries’ long term time series of number of fishers, and consequently total catches, calculated using data collected from a MS.

Using the definition of symbols presented in Table 1.1, the thesis is organized as follows.

Chapter 2 presents the notation, definitions, and problem setting used throughout this research. A literature review of the methods used to reduce the impact of nonresponse bias inherent in data used to estimate the proportion of the population ($\gamma$) that has a particular characteristic $x$ is presented.

In Chapter 3 the estimated parameters required for calculating total catch estimates are calculated and compared using the data collected from the MS and PDS for the various seasons. Assuming that the PDS method is more accurate than the MS, this comparison concludes that people who fished have a higher probability of returning the survey ($\pi_{x=t}$) than someone who did not ($\pi_{x=t}$) i.e. $\pi_{t} > \pi_{\bar{t}}$. Furthermore, it is
Table 1.1 Definitions of symbols used throughout Summary and Overview.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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| $\gamma$ | The probability that a member in the studied population has characteristic $x$.
| $\hat{y}_{\alpha}$ | The $y$ estimator for estimating $\gamma$ where $y \in \{e = \text{expectation}, m = \text{multinomial}, o = \text{ratio}\}$.
| $\pi_x$ | The probability that a member with characteristic $x$ will return the survey form.
| $\pi_{\bar{x}}$ | The probability that a member without characteristic $x$ will return the survey form.
| $p_{t,v}$ | The proportion of the licensed population that fished in season $t$ using fishing method $v \in \{\text{pot, dive}\}$.
| $F_{t,v}$ | The average retained catch per fisher using fishing method $v \in \{\text{pot, dive}\}$ in season $t$ given the fisher used fishing method $v$.
| $L_{t,v}$ | The number of licensees in season $t$ that fished using method $v \in \{\text{pot, dive}\}$.
| $\hat{\zeta}$ | The estimate of parameter $\zeta$.
| $r_{t,v}$ | The proportion of the population that fished in season $t$ using fishing method $v \in \{\text{pot, dive}\}$ that were ‘avid’ (fished at least a given number of days) for fishing method $v$.

seen that those fishers who reported having fished more frequently (‘avid’) were more likely to return the survey than those who reported a lower level of fishing effort (‘non-avid’). These results are consistent with surveys of other fisheries (Brown & Wilkins 1978; Fisher 1996) and fields of study (van Kenhove et al. 2002) that report highly ‘interested’ or ‘involved’ people are more likely to return a survey form and explains why the MS of people licensed to fish recreationally for rock lobster, produces much higher total catch estimates than the corresponding PDS.

The ratio estimator ($\hat{\gamma}_o$), which is traditionally applied to the collected MS data to estimate $\gamma$, assumes $\pi_x = \pi_{\bar{x}}$. Motivated by reducing the impact of biases such as those identified in Chapter 3 (i.e. $\pi_x > \pi_{\bar{x}}$), Chapter 4 constructs two alternative estimators to $\hat{\gamma}_o$, the multinomial ($\hat{\gamma}_m$) and expectation ($\hat{\gamma}_e$), by assuming different
values for $\pi_x$ and $\pi_{\bar{x}}$. Based on root-mean-square-error (RMSE), a well used criterion in the literature for comparing competing estimators, the ranges over $\gamma$ for which each of these estimators are best is determined. When $\pi_x > \pi_{\bar{x}}$, it is shown that on average, $\hat{\gamma}_c$ produces less biased estimates than $\hat{\gamma}_o$ for all values of $\gamma$. Further, if $\gamma \leq 0.65$ is also true, then $\hat{\gamma}_m$ produces even more accurate estimates again but at the risk of being more biased than $\hat{\gamma}_o$ if this condition does not hold. Methods for constructing confidence intervals for $\hat{\gamma}_c$, $\hat{\gamma}_o$, and $\hat{\gamma}_m$ are presented and simulations used to determine the true coverage rate of these intervals to include that estimator’s expected value, as well as $\gamma$ itself.

When $\pi_x > \pi_{\bar{x}}$, $\hat{\gamma}_c$ is less biased than $\hat{\gamma}_o$ over all $\gamma$. Given that $\hat{\gamma}_m$ can produce greater reductions in bias than $\hat{\gamma}_c$, but at the cost of being significantly more biased than even $\hat{\gamma}_o$ when $\gamma > 0.65$, Chapter 5 uses the time series of western rock lobster MS data to demonstrate choosing between $\hat{\gamma}_c$ and $\hat{\gamma}_m$ in a real situation where $\gamma$ is unknown. For each season $t$ and fishing method $v$ (potting or diving), the total catch equation for the western rock lobster fishery is formulated in terms of the proportion of licensees that fished for lobsters ($p_{t,v}$) and the proportion of those fishers who are avid ($r_{t,v}$). A process of deduction is presented and determines that $\hat{\gamma}_m$ is appropriate for estimating $p_{t,v}$. The PDS is used to define avidity so that using $\hat{\gamma}_m$ is appropriate for estimating $r_{t,v}$ in this study. The resulting estimates of number of fishers ($\hat{\tilde{L}}_{t,v}$ - a function of $\hat{p}_{t,v}$) and the average retained catch for each of these fishers ($\hat{\tilde{F}}_{t,v}$ - a function of $\hat{r}_{t,v}$) made using the MS data is then compared to those from the PDS to gauge the effectiveness of using $\hat{\gamma}_m$ rather than $\hat{\gamma}_o$ to estimate $p_{t,v}$ and $r_{t,v}$. The results of these comparisons are discussed.

In Chapter 6 the series of total catch estimates for each fishing method using $\hat{\gamma}_o$ and $\hat{\gamma}_m$ are modelled. A time series and an exponential model are used to model $\hat{\tilde{L}}_{t,v}$ and $\hat{\tilde{F}}_{t,v}$, respectively. Within fishing method, it is seen that the models for each of these estimates under $\hat{\gamma}_o$ and $\hat{\gamma}_m$ only differ by a scaling factor. Between fishing methods, it is seen that the optimal predictors for each model are different for potting and diving. The possible reasons for this, as well as possible implications for
management, are discussed in terms of the dynamics of the western rock lobster fishery.

This thesis is concluded in Chapter 7 with a general discussion on how well the objectives of the research have been met and identifies possible work that could be completed in the future to progress it further. Appendices are included at the back of this thesis and include details such as the MS forms used for collecting data from the recreational western rock lobster fishery, various algorithms and proofs, and other technical details that may be of interest to the reader.
Chapter 2

Definitions and Literature Review

In Chapter 1 the various survey methods typically used to estimate recreational catch and effort were discussed. Each of these methods have their own strengths and a combination of these methods can be used in a mixed survey design to optimize the precision of catch rate and effort information, both required to estimate total catch, subject to resource constraints. The mail survey (MS) method is cost efficient for estimating effort, especially in a diffuse fishery over a large spatial scale such as the western rock lobster recreational fishery. This data collection method however, can suffer from nonresponse bias and hence, effort estimates calculated from the returned survey forms may need to be adjusted to account for this bias.

In this Chapter the various methods in the literature used to reduce the affect of nonresponse bias on estimates calculated from collected MS data are presented. Definitions used throughout, a description of the survey setting for which these are defined, and details of the western rock lobster fishery that will aid understanding of the research to follow, are also presented.

2.1 Background

Various survey methods are conducted to estimate the distribution of various population characteristics that are unknown. The MS is one such method and requires sending a survey form to a random sample of the study population via post and then analysing the returned responses. The use of MSs is favoured in research on recreational fisheries where a succinct reference frame exists, due to its ease of implementation and lower cost compared to other survey sampling methods (Fox et al., 1998).
Different survey methods have their own specific sources of error (Cochran 1977; Lohr 2010). The four main sources of error inherent to the MS method are: sampling error – variation of the sample estimate as compared to the true population value due to the sampling design and chosen sample size; non-coverage error - exclusion of a subset of the population due to the chosen sampling frame; measurement error – inability to recall accurately or deception of respondents to accurately report accurate responses; and nonresponse error – the propensity of a person to respond is dependent on the population characteristic/s being measured by the survey.

Nonresponse bias is of great concern when it comes to making estimates from a collection of returned survey forms (Brown 1991; Strayer, et al. 1993; Stein, et al. 1999). If nonrespondents and respondents differ in terms of the population characteristic being measured then using the ratio estimator ($\hat{\gamma}_o$), which assumes respondents to be representative of the whole population, will produce a biased estimate of that statistic (Scheaffer et al. 1990). The issue of nonresponse has generally been regarded as much more of a concern with MSs than other methods because of the tendency for them to have lower response rates (Stein, et al. 1999).

Estimating total recreational catch using a MS method partly entails estimating the proportion of the surveyed population that fished. This research is focussed on identifying alternate estimators to $\hat{\gamma}_o$ that better estimate such proportions in the presence of nonresponse bias.

### 2.2 The Western Rock Lobster Fishery

This section gives a brief summary of relevant information regarding the western rock lobster (Panulirus cygnus) fishery to create a setting and assist in the understanding of parts of this research. For a comprehensive review the reader is referred to de Lestang et al. (2011).

Western rock lobsters are distributed along the West Australian coast from latitude 21°44'S to 34°24'S and are found in water as shallow as 1m and as deep as 200m (Figure 2.1). Western rock lobster supports the largest single species commercial
fishery in Australia and an important recreational fishery. In the last decade, seasonal commercial catches have ranged from about 5 500 to 14 000 t.

Figure 2.1: Western rock lobsters are found along the West Australian coast between latitudes 21°44′S and 34°24′S and can be found in water as shallow as 1m and as deep as 200 m (blue region). Depending on their concession, commercial fishers are restricted to fishing in one of three zones; A (Abrolhos Islands), B and C. The current sites for puerulus collector sites (de Lestang et al. 2011) are highlighted in red.
worth approximately A$200-400 million per annum. Further, up to 47,000 people in any one season have purchased a recreational licence to allow them to participate in the 7.5 month season (currently 15 November – 30 June).

The commercial sector fish using rock lobster pots set with a variety of baits and boats well capable of fishing the deeper waters of this fishery. The commercial fishery is divided into zones A (Abrolhos), B and C (Figure 1.1) and commercial fisherman are restricted to fishing the number of pots for which they are licensed to use in that zone.

A recreational fishing licence, which covers all licensed recreational fisheries, was required to fish for rock lobster using pots at the start of 1964. By the end of 1969, anybody fishing for rock lobster using any method was required to be licensed. In 1986, a recreational licence specific to rock lobster fishing was introduced and in 1992 an ‘umbrella’ licence was also available to purchase, at a discounted rate, which gave the option to fish rock lobster and other licensed recreational fisheries (abalone, marron, freshwater and netting). Prior to the introduction of the umbrella licence, licences would expire at the end of the coming June. The introduction of the umbrella licence saw all of these licences expire 12 months from the date of purchase.

Unlike the commercial, recreational fishers are not restricted to fish in a specific zone but due to the remoteness of the Abrolhos Islands and other coastal towns, recreational fishing mainly occurs off the mainland coast from Augusta to Kalbarri (Figure 2.1). Typically completed from a boat, the two primary fishing methods available to recreational fishers are potting and diving. Recreational fishers generally have access to much smaller and less rugged boats than commercial fishers which tends to restrict their fishing to well-protected waters in depths less than 40 m.

As described in de Lestang et al. (2011), western rock lobsters first appear on the inshore reefs of the West Australian coast as puerulus (the post-larval stage) one year after they are hatched from eggs in the deeper waters off the coast. Through a series of moults these puerulus grow to become juvenile rock lobsters and feed and continue to grow on the shallow onshore reefs for the next three or four years. At
approximately four years of age these lobsters moult from their typical dark red shell into a paler coloured ‘whites’ shell. Before reverting back to their typical colour at their next moult, most of these animals migrate west to water up to 200 m deep about 100 km off the coast in what is referred to as the ‘whites’ migration that commences in late Spring each year. These animals remain in those waters for the remainder of their lifecycle, including breeding.

The abundance of puerulus settling on the inshore reefs each year has long been measured by the use of collectors which are designed to provide artificial habitat to puerulus as well as being easy to remove from the water in a dinghy. These collectors are fixed in shallow waters of well-protected inshore reef areas in strategic sites along the coast. At every full moon each of these collectors is removed from the water, the residing puerulus removed and counted, and then placed back to their previous position. Each site typically has 6 collectors and the average count of puerulus collected by each of these is calculated and then summed over all the full moons during the period of May to the following April, to determine the index of puerulus settlement at that site for that season. The peak period of settlement is August to January.

Using the time series of puerulus indices for the various collector sites, the interannually fluctuating commercial catches of western rock lobster up and down the coast, are reliably explained by puerulus settlement 3 and 4 years earlier (Morgan et al. 1982; Phillips 1986; Caputi et al. 1995). These relationships generally indicate that the majority of western rock lobsters enter the fishery as legally sized animals at approximately 3 and 4 years post settlement. Melville-Smith et al. (2001) and (2004) has shown that puerulus settlement at Alkimos (Figure 2.1), a location just north of the Perth metropolitan area where the majority of recreational fishing occurs, is well correlated with the recreational lobster catches 3 and 4 years later.

The reliable prediction of catches for the commercial sector has allowed management to be proactive in implementing stock protective strategies using input control techniques on both the commercial and recreational sectors over time (de Lestang et al. 2011). Management controls used have included restrictions on pot designs, minimum size rules, maximum size rules on females and protection of females
carrying eggs or with ovigerous setae. In addition to these regulations, the recreational sector has also been limited to using a maximum of two pots per licence holder, four pots per boat with two or more licence holders on board, and varying daily bag, boat and possession limits. There has been a relaxation of some of these regulations since the end of the 2010/11 season.

The western rock lobster fishery has undergone a formal process to allocate catch shares to the recreational and commercial sectors (Crowe et al. 2013). This process led to proportions of the total sustainable catch allocated to the commercial and recreational sectors being set at 95% and 5%, respectively. This Integrated Fisheries Management (IFM) arrangement includes an expectation that the catch for each sector will be known so that management adjustments can be made to regulate catch, if required, to maintain these proportions. The total catch of the commercial sector is determined from mandatory records (under the FRMA act) of catches as part of the licensing agreement and processor returns are also available to validate the catch. The recreational sector however, is not legally required to report their individual catches so an end of season MS has been used since 1986/87 to estimate the total catch for this part of the fishery. A PDS has been has also been run in recent seasons (2000/01, 2001/02, 2004/05 – 2008/09) to assess the reliability of these MS estimates (IFAAC 2007).

2.3 The Survey Setting

MSs have been used extensively to estimate the retained catches of various recreational fisheries. The implementation of this survey method will differ to account for the many differences that can exist between fisheries such as timing of their seasons and numbers and types of fishing methods used. We will discuss the MS problem in terms of the western rock lobster fishery. This is a licensed multi method fishery and its methodology for summarizing the returned survey form information to estimate the total catches should have similarities with other fisheries.

A first step in running a MS is to identify the reference frame of names and addresses of people (sampling units) able to participate in the fishery. In a licensed fishery such
as the western rock lobster fishery the licensing database provides this reference frame. For fisheries where people do not need to be licensed, then a list of names and addresses within a defined geographical area that includes all those people who have access to the fishery is gathered from other sources such as telephone books, electoral rolls and school enrolments. From the number of people in the identified reference frame for season $t$ (of size $N_t$), $n_t$ are randomly selected and sent a survey form. In practice, due to say a change of address, some of these forms will not be received by the intended people and may be ‘Returned to Sender’. It will be assumed throughout that $n_t$ has been adjusted for the number of such returned survey forms.

For the purposes of estimating total catch, a person is considered to have ‘returned the survey’ if the survey form is received by the designated collection agency and it identifies whether or not the survey respondent fished or did not fish. In a multi method fishery such as the western rock lobster fishery, the survey form is designed to ask if the person fished at least once for rock lobster in the recently completed season and if they did, whether they used the methods of potting &/or diving. The survey form then asks how many days they fished using each method and how many lobsters they retained. Historically, this survey form has been returned reporting having fished for rock lobster but providing no further information about how and what they caught. Other survey forms have been returned reporting they fished and what methods they actually used but no information about how often they fished or what they caught. In the complete response, which is the usual case, if a person has reported they fished, they have also provided all further details about that fishing.

The returned survey forms are summarized into the number returned ($n_{t,R}$), the number of these reporting having fished ($n_{t,G}$), and the number of these that reported potting ($n_{t,\text{pot}}$) and diving ($n_{t,\text{dive}}$). To deal with incomplete responses where a number of people report they fished but not what methods they used ($n_{t,f,-}$), $n_{t,\text{pot}}$ and $n_{t,\text{dive}}$ are weighted up by $n_{t,G}/n_{t,f,-}$. Doing this assumes that a person not providing this information is independent of what fishing method(s) they used. The returned survey forms are also used to estimate the average catch retained for a fisher in season $t$ using fishing method $v \in \{\text{pot, dive}\}$ ($\hat{P}_{t,v}$), given they fished using
fishing method \( v \), and is also assumed representative of the people who returned the survey form but did not report this information.

Using the relevant summary data to estimate the proportions of the licensed population that potted or dived (\( \hat{p}_{t,v} \), \( v \in \{\text{pot}, \text{dive}\} \)), the estimated total catch for each of these fishing methods (\( \hat{T}_{t,v} \)) is then calculated by multiplying the number of licensees estimated to have fished (\( \hat{N}_{t,v} \)) by the average retained catch of someone reported using that method (\( \hat{F}_{t,v} \)).

The ratio estimator (\( \hat{\gamma}_o \)) is commonly used to estimate \( p_{t,v} \). Using data from a MS, the \( \hat{\gamma}_o \) estimates the proportion of the population that has some characteristic \( x \) (e.g. fished) as the proportion of survey returns that reported having \( x \) (\( n_x \)) compared to not (\( n_{\overline{x}} \)) i.e. \( \hat{\gamma}_o = n_x / (n_x + n_{\overline{x}}) \). This estimate is unbiased if the probability of returning the survey form for someone with characteristic \( x \) (\( \pi_x \)) is the same as someone without (\( \pi_{\overline{x}} \)) i.e. \( \pi_x = \pi_{\overline{x}} \). When \( \pi_x \neq \pi_{\overline{x}} \), then various methods have been applied in the literature to adjust \( \hat{\gamma}_o \) post hoc in an attempt to correct for its resulting bias. Under the assumption that \( \pi_x > \pi_{\overline{x}} \), the objective of this research is to identify estimators that are less biased than \( \hat{\gamma}_o \) to reduce the need to apply some correction method to the resulting estimates.

### 2.4 Notation and Terminology

A raft of definitions and symbols will be used throughout this research. Without loss of generality to other recreational fisheries, definitions are presented with the recreational western rock lobster fishery and its employed survey methods, in mind. In light of this, definitions will be presented in terms of the primary fishing methods for this fishery of potting and diving, which differ in terms of their catch rates and effort levels. This section presents the notation and terminology required and used throughout to define and solve the general problem presented in Section 2.3:
Chapter 2

Subscripts/Superscripts

\( k \)  
The licence type:  
\( k \in \{ R = RL \text{ specific}, U = Umbrella \} \)

\( s \)  
The survey method used to collect data:  
\( s \in \{ m = Mail, pds = PDS \} \)

\( t \)  
The fishing season. For the recreational western rock lobster fishery this is defined as 15 November in year WXYZ through to 30 June in the following year WXYZ+1. In tables, season will be labelled as WXYZ/(YZ+1) e.g. 1999/00. As labels on axes, season will be labelled as YZ/(YZ+1) e.g. 99/00. As labels on data points in graphics, season will be labelled as YZ(YZ+1) e.g. 9900.

\( v \)  
The method used for fishing:  
\( v \in \{ \text{pot, dive} \} \)

\( y \)  
The estimator type used to estimate the proportion of the population that has a dichotomous characteristic:  
\( y \in \{ e = \text{expectation}, m = \text{multinomial}, o = \text{ratio} \} \).

Terms

Estimator  
A rule for calculating an estimate of a given quantity based on observed data.

Fishery  
Is an entity engaged in harvesting fish.

IFM  
Integrated fisheries management – a process that manages access to the ‘resource’ by all stakeholders such that their combined exploitation of the resource is managed in an integrated way.

Non-respondent  
A person who does not return the survey form or does but did not report whether or not they fished.
Recreational  
Is an activity of leisure (time spent away from business, work and domestic chores).

Respondent  
A person who returns a survey form and indicated whether or not they fished.

Survey  
A methodology used to collect information regarding a particular population.

Distributions

Binomial  
\[ B(Z; m, q) = \binom{m}{z} q^z (1 - q)^{m-z} \quad \text{for} \quad 0 \leq z \leq m; \quad \text{else,} \\
B(Z; m, q) = 0; \quad \text{where} \quad 0 \leq q \leq 1. \]

Indicator  
\[ I(x) = 1 \text{ if condition } x \text{ is true and } I(x) = 0, \text{ otherwise.} \]

Multinomial  
\[ M(Z; m, Q) = \frac{m!}{z_1!...z_k!} q_1^{z_1} ... q_k^{z_k} \quad \text{for} \quad z_1 + ... + z_k = m; \quad \text{else,} \\
M(Z; m, Q) = 0; \quad \text{where} \quad Z = (z_1, z_2, ..., z_k) \text{ contains all non-negative integers and} \quad Q = (q_1, q_2, ..., q_k) \text{ contains non-negative real numbers such that} \sum_{i=1}^{k} q_i = 1. \]

Snedecor F  
\[ F(Z; v_1, v_2) = \frac{\Gamma((v_1+v_2)/2)\Gamma(v_1/2)\Gamma(v_2/2)}{\Gamma((v_1+v_2)/2)(v_1+v_2)/2)\Gamma(v_1/2)\Gamma(v_2/2)} \quad \text{for} \\
0 < z < \infty; \quad \text{else,} \quad F(z; v_1, v_2) = 0; \quad \text{where} \quad \Gamma(\beta) = \int_0^\beta x^{\beta-1}e^{-x}dx \quad \text{and} \quad v_1 \text{ and } v_2 \text{ are positive integers known as the degrees of freedom.} \]

Uniform  
\[ U(h_0, h_1) = \frac{1}{h_1-h_0} \text{ for} \quad h_0 < z < h_1; \quad \text{else,} \quad U(h_0, h_1) = 0. \]

Cbinom(z_1; m, q)  
The cumulative probability for \(0 \leq z \leq z_1\) where \(Z \sim B(z; m, q)\)  
i.e. \(\sum_{z=0}^{z_1} B(z; m, q)\).
Chapter 2

\( \text{qbinom}(r; m, q) \)  The quantile at which the cumulative probability for 
\( 0 \leq z \leq \text{qbinom}(r; m, q) \), where \( Z \sim B(z; m, q) \), is equal to \( r \).

\( \text{rprob}(1, g_0, g_1) \)  A number between \( g_0 \) and \( g_1 \) chosen randomly such that all possible values have equal chance of being selected.

**Parameters**

\( a_v \)  Avidity definition - a person is categorized as ‘avid’ for fishing method \( v \) if they fish > \( a_v \) days for the season; else, ‘non-avid’.

\( \alpha \)  Probability of a type I error.

\( \text{df} \)  Degrees of freedom.

\( N_t^{(s)} \)  The size of the population which is being randomly sampled by survey method \( s \) in season \( t \).

\( n_t^{(s)} \)  The number of people randomly selected to receive a survey form for survey \( s \) in season \( t \). It is assumed that this number has been adjusted for ‘Return to Sender’.

\( \mu_Z \)  The expected value of variable \( Z \).

\( \pi_t^{(s)} \)  The probability that a person in the population with characteristic \( x \) will respond to survey type \( s \) in season \( t \).

\( \pi_t^{(s)} \)  The probability that a person in the population that does not have characteristic \( x \) will respond to survey type \( s \) form in season \( t \).

\( \sigma_Z^2 \)  The population variance of \( Z \).

\( \sigma_{Z_1, Z_2} \)  The covariance between \( Z_1 \) and \( Z_2 \).
Variables

\( b_\gamma(\widehat{\gamma}) \)  
The bias of estimator \( \widehat{\gamma} \) in estimating \( \gamma \) is defined as
\[
b_\gamma(\widehat{\gamma}) = \mathbb{E}[\widehat{\gamma} - \gamma].
\]

\( CVr_\gamma(\nu, \alpha) \)  
The probability that a \( 100(1 - \alpha)\% \) confidence interval for estimator \( \widehat{\gamma}_y \) includes the parameter \( \nu \) i.e.
\[
Pr(L_\alpha(\widehat{\gamma}_y) \leq \nu \leq U_\alpha(\widehat{\gamma}_y)).
\]

\( \widehat{\gamma}(s) \)  
The proportion of respondents that reported the catch and effort for fishing method \( \nu \), that reported fishing method \( \nu \) for \( > \alpha_s \) days.

\( \widehat{F}(s) \)  
The average retained catch reported by people in survey \( s \) of season \( t \) caught using fishing method \( \nu \) given they reported their catch and effort for this method.

\( \widehat{F}(s) \)  
The average retained catch reported by people in survey \( s \) of season \( t \) caught using fishing method \( \nu \), given they reported using this method and provided the catch and effort information for this method, making use of the avidity definition \( \alpha_s \), i.e.
\[
\left(1 - \widehat{\gamma}_t^*(s)\right) \widehat{F}_t^{(s)} + \widehat{\gamma}_t^*(s) \widehat{F}_t^{(s)}.
\]

\( \widehat{F}(s) \)  
The average retained catch reported by people in survey \( s \) of season \( t \) caught using fishing method \( \nu \), given they reported using this method for \( > \alpha_s \) days and provided their catch and effort information for this method.

\( L_\alpha(\widehat{\gamma}) \)  
The lower limit of the \( 100(1 - \alpha)\% \) confidence interval for estimator \( \widehat{\gamma} \).

\( \widehat{F}(s) \)  
The average retained catch reported by people in survey \( s \) of season \( t \) caught using fishing method \( \nu \), given they reported using this method for \( \leq \alpha_s \) days and provided their catch and effort information for this method.
\( \hat{N}_{t,v,g_p}^{(s)} \) The estimated number of licensees in season \( t \) that fished using fishing method \( v \) i.e. \( \hat{N}_{t,v,g_p}^{(s)} \).

\( \hat{\mu}_Z \) The estimated population mean of variable \( Z \).

\( n_{t,R}^{(s)} \) The number of respondents to survey \( s \) in season \( t \). A person is considered to have returned the survey if they report having fished or not, independent of reporting which methods they used i.e. \( n_{t,R}^{(s)} \).

\( n_{t,f}^{(s)} \) The number of respondents to survey \( s \) in season \( t \) that reported having fished.

\( n_{t,\bar{f}}^{(s)} \) The number of respondents to survey \( s \) in season \( t \) that reported not having fished.

\( n_{t,f,-}^{(s)} \) The number of respondents to survey \( s \) in season \( t \) that reported having fished but not the fishing methods used i.e. \( n_{t,f,-}^{(s)} \leq n_{t,f}^{(s)} \).

\( n_{t,v}^{(s)} \) The number of respondents to survey \( s \) in season \( t \) that reported using fishing method \( v \), weighted up by \( n_{t,f}^{(s)}/n_{t,f,-}^{(s)} \) to account for those people who reported having fished but not the fishing methods used.

\( n_{t,\bar{v}}^{(s)} \) The number of respondents to survey \( s \) in season \( t \) that are assumed to not have fished using fishing method \( v \) i.e. \( n_{t,\bar{v}}^{(s)} = n_{t,R}^{(s)} - n_{t,v}^{(s)} \).

\( n_{t,v,a}^{(s)} \) That part of \( n_{t,v}^{(s)} \) estimated to have fished using fishing method \( v \) for \( > a_v \) days. This estimate accounts for respondents that reported fishing methods used but not catch and effort i.e. \( n_{t,v,a}^{(s)} \).
\( n_{t,v,\bar{a}}^{(s)} \) That part of \( n_{t,v}^{(s)} \) estimated to have fished using fishing method \( v \) for \( \leq a_{v} \) days i.e. \( n_{t,v}^{(s)} - n_{t,v,a}^{(s)} \).

\( \hat{P}_{t,v,u}^{(s)} \) The proportion of the surveyed population that fished in season \( t \) using fishing method \( v \) estimated using estimator \( \hat{\gamma}_{u} \) i.e. \( \hat{\gamma}_{u} \left( w = n_{t}^{(s)} , w_{x} = n_{t,v}^{(s)} , w_{x} = n_{t,v,\bar{a}}^{(s)} \right) \).

\( P \) Probability of a type I error.

\( \hat{\rho}_{x,y} \) Estimated correlation between variables \( x \) and \( y \).

\( \hat{\gamma}_{y}^{(s)}_{t,v,y_{0},y_{a}} \) The proportion of the surveyed population that fished in season \( t \) using fishing method \( v \), that were ‘avid’, estimated using estimator \( \hat{\gamma}_{y} \) i.e. \( \hat{\gamma}_{y} \left( w = n_{t}^{(s)} \hat{P}_{t,v,y_{0},y_{a}}^{(s)} , w_{x} = n_{t,v,\bar{a}}^{(s)} , w_{x} = n_{t,v,\bar{a}}^{(s)} \right) \).

\( \text{RMSE}(\hat{\theta}|\theta) \) Root-mean-square-error is the root of the sums of squared errors of the \( j \) observations of the estimator \( \hat{\theta} \) from the true value of the parameter being estimated \( (\theta) \) i.e. \( \sqrt{\frac{1}{J} \sum_{i=1}^{J} (\hat{\theta}_{i} - \theta)^{2}} \).

\( U_{\alpha}(\hat{\gamma}) \) The upper limit of the \( 100(1 - \alpha)\% \) confidence interval for estimator \( \hat{\gamma} \).

\( \hat{\sigma}_{Z}^{2} \) The estimated population variance of variable \( Z \).

\( \hat{T}_{t,v,y_{0}}^{(s)} \) The estimated total retained catch of the surveyed population in season \( t \) using fishing method \( v \) where the proportion of the population who fished using this method was estimated using \( \hat{\gamma}_{u} \) i.e. \( \hat{T}_{t,v,y_{0}}^{(s)} = \hat{P}_{t,v}^{(s)} \hat{T}_{t,v}^{(s)} \).
Chapter 2

\[ \hat{R}^{(s)}_{t,v,y_0,y_a} \] The estimated total retained catch of the surveyed population in season \( t \) using fishing method \( v \) where the proportion of the population who fished using this method was estimated using \( \hat{y}_{y_0} \) and the proportion ‘avid’ by \( \hat{y}_{y_a} \) i.e. \( \hat{R}^{(s)}_{t,v,y_0,y_a} = \hat{R}^{(s)}_{t,v,y_0,y_a} \).

\[ \hat{R}^{(s)}_{t,y_0,y_a} \] The estimated total retained catch of the surveyed population in season \( t \) using all fishing methods i.e. \( \sum_v \hat{R}^{(s)}_{t,v,y_0,y_a} \).

2.5 Minimizing Nonresponse Bias

The two general approaches for minimizing the affect of nonresponse on parameter estimates resulting from a set of returned survey forms are: i) adjusting the survey design to maximize the return rate of survey forms; and ii) suspecting that this bias is an issue, apply mathematical methods to adjust the estimates post-hoc, to reduce nonresponse bias.

i) Maximizing return rates of the survey

In an attempt to control the nonresponse bias, a common approach is to maximize the return rate (100\% \( \% \)) of the survey form. If a return rate of 100\% can be achieved, then nonresponse bias does not exist. When the return rate is less than 100\% however, Groves (2006) argues that the return rate is not necessarily a true indication of nonresponse bias. Nonresponse bias is related to the covariance between the variable being measured by the survey and the response propensity for the sampling unit given their value for that variable. This covariance, Groves argues, is not related to return rates. Still, as discussed later, increasing the return rate can have the benefit of narrowing the possible range on the population parameter being estimated (Manski 1995); particular when estimating the proportion of the surveyed population (\( \gamma \)) that has some characteristic \( x \).

To increase return rates of surveys, various techniques have been employed: financial and material incentives (e.g. entry to a prize draw for returning the survey), personalized covering letter, questionnaire design (e.g. survey length, colour of
The TDM “posits that questionnaire recipients are most likely to respond if they expect that the perceived benefits of doing so will outweigh the perceived costs of responding” (Dillman 1991). This requires reducing respondent burden (e.g. making the survey easier and quicker to complete), increasing perceived rewards (e.g. making the survey interesting and relevant to the recipient), and increasing trust (e.g. using official stationary) (Dillman 1978 & 1983). After the initial posting of the questionnaire that has been developed to be ‘appealing’ and ‘engaging’ to the survey recipient, a series of ‘follow-up’ procedures are carried out at various time points to encourage people to complete the survey if they have not yet done so (e.g. sending reminder postcards, sending another copy of the survey in case they have misplaced the original).

The return rate is not a definitive indicator of whether or not the returned survey forms are biased by nonresponse. A survey that has a return rate of 50% can suffer more from nonresponse bias than another survey that say, has a return rate of only 20%. Wilson (1999) argues however that a low response rate should make you question the quality of the survey method in how carefully the research was executed.

To gauge the extent to which nonresponse may affect the parameters of interest Manski (1995) suggested constructing Manski Bound’s, which are the bounds on the parameter estimate as if the survey had a 100% return rate. To do this, the lower limit is calculated as the parameter estimate resulting from the respondent values along with assuming the minimum possible value for each of the nonrespondents. The upper limit is calculated similarly but assuming the maximum possible value for each of the nonrespondents. In the case of a dichotomous variable (e.g. did you fish? Yes/No), this equates to determining the lower limit when all nonrespondents take on the value zero(0) (e.g. no) and the upper limit when all nonrespondents take on the
value one (e.g. yes). To illustrate, consider a survey that is designed to estimate the proportion of people in the surveyed population that fished ($p_t$). Having sent $n_t$ survey forms, $n_{\ell}$ and $n_{\bar{\ell}}$ survey forms are returned saying they did and did not fish, respectively. To determine the lower bound on $p_t$ it is assumed that all of the $n_t - n_t - n_{\bar{\ell}}$ nonrespondents did not fish and hence, $p_t \geq (n_{\ell} + 0)/n_t$ or $p_t \geq n_t/n_t$.

The upper bound is calculated assuming all nonrespondents fished and hence, $p_t \leq (n_t + [n_t - n_t - n_{\bar{\ell}}])/n_t$ or $p_t \leq (n_t - n_{\bar{\ell}})/n_t$. Therefore, the possible range on $p_t$ if all survey recipients eventually responded is $n_t/n_t \leq \hat{p}_t \leq (n_t - n_{\bar{\ell}})/n_t$.

Having constructed Manski’s Bound some assessment can be made as to whether or not the return rate is sufficient to meet the particular requirements of the study. In a heuristic approach, if the range of the bound is considered too broad, then this would indicate further follow-up surveys to increase the response rate to further reduce the Manski’s Bound on the estimate. In the case of estimating the mean of a dichotomous variable such as $p_t$ this method is easily applied given nonrespondents can take on only one of two values i.e. 0 or 1. For a parameter that is measured on a continuous scale, such as the retained catches of a fisher in a season, this might not be as easily applied. The minimum value for the retained catch of a nonrespondent fisher may be safely assumed zero (0) but there may be several options, including levels greater than those actually reported by respondents, that are all feasible for the maximum e.g. nonrespondent fishers might tend to be those who caught more fish than those responding.

Viswesvaran et al. (1993) takes assessing the adequacy of the achieved response rate a step further than Manski’s Bound by determining a “critical response rate”. Similar to using a t-test for determining if a sampled mean is statistically similar to some critical value $y_c$ (this could be the sampled mean itself or its 95% lower or upper confidence limit), the required mean response of all nonrespondents to reverse the conclusion of the returned samples is determined and the current return rate is concluded sufficient if this required mean response is regarded as infeasible. This judgement of ‘infeasibility’ requires a guessimate of the nonresponse value which can either be made by judgment or an imputation procedure (see following section).
If a nonresponse bias exists then it has a direction i.e. the true population parameter is either over or under estimated. Identifying the direction of this bias has the benefit of reducing the width of Manski’s Bound by including this direction as a restriction on the bound, as opposed to the cost of having to further increase the return rate of the survey form. The direction of nonresponse bias can be identified using ‘extrapolation’.

Extrapolation methods make use of trends in the time-to-return of surveys to determine the direction of nonresponse error (Armstrong & Overton 1977; Jackman 1999). Using one wave of sampling, Ferber (1949) suggested studying whether there is a trend in the distribution of replies to particular survey questions with respect to the time at which the survey is returned. If trends in these survey replies exist with time-to-return, then the existence and direction of nonresponse bias could be immediately concluded.

To increase the overall return rate from which conclusions are based, the extrapolation method of Ferber (1949) has been taken further by using the multiple follow-up procedures of the TDM. The respondents to each successive wave of sampling are often regarded as ‘reluctant’ and their responses to the survey questions are considered to more closely represent nonrespondents to the first wave (the initial survey post-out). This assumption has been used in many studies in an attempt to describe nonrespondents and to explain which people are more likely to return surveys (e.g. Woodside et al. 1984; Filion 1976).

In an attempt to determine if two waves of sampling are sufficient to determine the direction of bias, Armstrong & Overton (1977) studied the literature that reported more than two waves of sampling. By assuming the comparison of the parameter estimates between the first and third wave gives the true direction of the bias, an error rate for all these surveys was calculated by comparing this to the direction of bias indicated by comparing the first two waves. This study concluded that the mean response from the first two waves of sampling was useful in determining the direction of bias.
Armstrong & Overton (1977) note that the use of different stimulus for the different waves of sampling for the surveys in their study may have introduced further biases into their comparisons given that a person’s actual response to a question can be affected by the stimulus to arouse that response. On extrapolation methods in general, Groves (2006) notes that these methods provide little information about the nonresponse bias that remains after repeated follow-ups. It is possible that the remaining nonrespondents are completely different to ‘late’ respondents.

Having determined the direction of the bias, the width of the Manski Bound can be reduced without the expense of additional inducements to increase response rates. To demonstrate, assume that for a surveyed population that it is concluded that people who fished are more likely to return the survey form than those that did not i.e. \( n_f/n_t > p_t \). This leads to the traditionally used ratio estimator \( \hat{\gamma}_o \) producing an estimate of the proportion of people in the surveyed population that fished \( \hat{p}_o \) that is greater than the true population parameter \( p_t \) (i.e. \( \hat{\gamma}_o > p_t \) since \( \hat{\gamma}_o \) estimates \( p_o \) as the proportion of respondents who reported having fished. For this example it was previously shown that \( n_f/n_t \leq p_t \leq (n_t - n_f)/n_t \). Given that \( \hat{\gamma}_o > p_t \), this bound can be reduced to \( n_f/n_t \leq p_t \leq \hat{\gamma}_o \).

### ii) Mathematical adjustments for reducing nonresponse bias on estimates

Having conducted the MS and suspecting that the resulting raw estimates are likely to be affected by nonresponse bias, various mathematical methods have been used in an attempt to remove this bias with varying levels of success. Generally, these methods adjust the summarized estimate of the variable being measured (e.g. probability that they fished for rock lobster) by attempting to account for the likely responses of the nonrespondents. These methods differ in how they account for these nonresponses.

One method for determining what the likely responses of nonrespondents would have otherwise been is to randomly sample a proportion of these people to determine their actual responses (‘double-sampling’ method). These responses can then be used to summarize the likely responses of all the nonrespondents to the first wave of
sampling (Hanson et al. 1993a; Hanson et al. 1993b). Double sampling was used successfully in a health study of Otsego County residents and used a MS for the first wave and a telephone survey to subsample some of the nonrespondents in a second wave (Jenkins et al. 2008). Armed with a census study for the same population, the use of this double sampling led to accurate estimates. If there are nonrespondents to the second wave of sampling however, this will not necessarily always be the case since nonresponse bias can still exist.

Assuming that there are no nonrespondents after two waves of sampling, Srinath (1971) proposed a method for determining the optimal initial sample size and subsampling fraction of nonrespondents in the second wave of sampling to minimize the cost of the overall survey for a desired level of precision in the variable being estimated. Even with more than two waves of sampling using a MS however, it is often the case that at a certain point the remaining nonrespondents are extremely reluctant and any further mail-outs are not useful. Attaining a 100% response rate to the second wave of sampling is therefore likely to require employing a more expensive sampling method such as multiple telephone interview contacts.

In an effort to reduce the cost of the more expensive survey method in attaining a 100% response rate, Srinath (1971) suggested expanding sampling to more than 2 stages where at each stage, a subsample of nonrespondents to the previous are again surveyed and the process is repeated until in the last stage, no nonresponses occur. It is at this last stage where more expensive survey methods are used to survey a smaller group of people that would otherwise be required after only two stages of sampling.

Using different survey methods as often required in the final wave of sampling where a 100% return rate is required, introduces the issue of ‘mode-difference’ where the way a person responds to a self-administered MS may be different to what they would report in say, an interviewer based telephone survey. A measure of difference between parameter estimates of the two survey methods can be obtained by interviewing a subsample of respondents to the MS with the ‘follow-up’ survey and comparing responses. The observed measure of difference can then be used to adjust the responses in the final wave for any bias due to a change in survey mode.
In studies where nonrespondents still exist after multiple waves of sampling have been used, depletion analysis type methods have been used to estimate what the likely average response would be if a 100% response rate had been achieved. In a MS of game bird hunters, Filion (1976) used a regression model of successive waves of sampling to attempt to remove nonresponse bias from key variables of the study. The survey used three waves of sampling where all nonrespondents to the previous wave were sent another survey form. After three waves of sampling there still existed nonrespondents. Plotting the mean response of all respondents up to and including each wave versus the cumulative return rate up until that wave for the variable being studied, a linear regression was fitted and used to predict what the estimate would be for that variable with a 100% return rate.

Such analysis’ as that by Filion (1976), which assume that people who respond later in time, or due to additional contacts, are more similar to the nonrespondents than the earlier respondents, is generally referred to in the literature as the “continuum of resistance”. In a telephone survey where a random sample of people with known responses was contacted multiple times until they agreed to participate in the survey, this was shown to not necessarily be a good assumption (Lin & Schaeffer 1995).

The impact of nonrespondents on the bias of a survey can not be directly assessed by the response rate. Nonresponse bias exists on some population variable being estimated directly from the collection of returned surveys if there is a covariance between that variable and response propensity (Groves 2006).

In a MS of people’s political opinions, Pearl & Fairly (1985) suggested the use of Kendall’s tau statistic (Kendall 1938) for testing the correlation between different reported opinions and each respondent’s reported level of “strength of opinion”. Studying the significance of the calculated Kendall tau statistic for two different surveys of the voting population prior to two different elections, it was seen that the nonsignificant result led to that survey predicting the election outcome whilst the election associated with the significant result, did not. The Kendall tau statistic might be used to test for significant correlation between response propensity and a
particular variable being measured as part of testing for likely existence of a nonresponse bias.

When nonresponse bias is assumed to exist, weighting and imputation methods have been widely used to adjust for this bias. These methods differ in how they account for the likely responses of nonrespondents; imputation methods substitute the nonrespondent data with imputed estimates and weighting methods weight respondents to adjust for their differential representation in the returned sampled compared to that in the initial surveyed sample. Both of these methods share the common requirement of needing the same auxiliary data (AD) for both respondents and nonrespondents.

Imputation methods are generally preferred to weighting methods to account for item nonresponse (i.e. survey returned but some certain questions not answered) and the later, for unit nonresponse i.e. the survey is not returned. This research is concerned with unit nonresponse and hence, this literature review focuses on weighting rather than imputation methods. A comprehensive review of the methodology and issues for various imputation methods can be found in Donders et al. (2006).

Based on AD that is available for all survey recipients (e.g. age, sex), weighting methods post-stratify survey respondents and nonrespondents into strata that are considered homogenous in regards to their response propensities and the survey variable under study. Little (1986), Eltinge & Yansaneh (1997) and Shaw et al. (2010) detail different methods that are generally used to determine these groupings. Weighting methods then weight-up the responses in each cell to account for the number of population units it represents given the number of nonrespondents in its strata.

There are many methods used to determine the weights to be applied to each strata and include the weighting class method, raking, inverse response propensity and calibration (e.g. Kalton 1981; Valliant 1993; Groves 2006; Särndal 2007; Sukasih et al. 2009). The inverse response propensity model is widely used given its advantage over other cell adjustment methods in that weightings from groupings based on multiple AD are more easily calculated by using a logistic regression.
In constructing classes, auxiliary data is used to identify if there are non-random reasons as to why people do not return surveys e.g. older people are more likely to return surveys. This difference however, does not prove for a given strata, that the would-be responses of nonrespondents are the same as respondents. Hence, weighting methods may just be adjusting for population representation and not a difference in how nonrespondents would differ in their response to the survey question under study.

Fisher (1996 & 1997) used response propensity stratification to adjust for nonresponse bias in angler MSs. In a study of policy performance for Dutch municipalities, where the performance measures for respondents and nonrespondents were also gathered from an independent source, van Goor & Stuiver (1988) showed that the adjusted estimate based on different weighting methods were no better than not adjusting at all. Applying such methods could then lead to false confidence in the data.

It has been shown that for AD to reduce nonresponse bias, whilst also not increasing the variance in the adjusted estimate, it must be correlated with both response propensity and the survey variable of interest (Little & Vartivarian 2005; Groves 2006; Kreuter et al. 2010). If the covariance between the two variables is caused by a common auxiliary variable that predicts both, then such methods are useful in reducing the nonresponse bias. If however, the covariance between the two variables is due to response propensity being directly influenced by the variable being measured (e.g. an anglers propensity to return the survey increases with increasing catch and not due to any auxiliary variable) then such methods are not likely to work well.

The requirement that the AD is correlated with response propensity can be easily established from the survey; the requirement that the AD is correlated with the variable being measured, given the responses of nonrespondents are unknown, is not. Under the assumption that the variance of each of the weights is homogenous, Kish (1965) shows that applying the weighting method using AD that is not correlated with the variable being studied increases the variance in the adjusted estimate.
compared to the unadjusted, by a ratio of the square of the coefficient of variation (cv) of the weights plus 1 (i.e. $cv^2 + 1$) without reducing the nonresponse bias.

The study by Kreuter (2010) showed examples where two predictors individually had strong correlations with response propensity and the survey variable under study and separately, led to adjusted estimates that were significantly different to the unadjusted. Using both predictors together however, resulted in an adjusted mean that was similar to the unadjusted. Kreuter & Olsen (2011) illustrates that this is to be expected when the correlation of each predictor are not in the same direction for both response propensity and the survey variable. Although the adjusted estimate is unchanged to the unadjusted, it is seen that the variance of the estimate was reduced. It is only when the predictors used are not related to the survey variable that the estimate is unchanged but the variance increased.

The study of Kreuter & Olsen (2011) demonstrates that adjusting the estimate for nonresponse bias when all the important AD have not been identified and used in defining the cells, may even inflate the level of bias in the resulting estimate.

Using auxiliary data (e.g. age, education) that was available for both nonrespondents and respondents, Page (1991) corrected for nonresponse bias in a MS by using a predictive model to establish the relationship between this background information and responses. These models were then used to impute values for nonrespondents before recalculating the statistics of interest.

Surveys are performed to estimate the unknown distribution of some characteristic of the population being sampled. With this being the case, it is impossible to know if strata are homogenous in terms of the variable being studied and hence, stratification is based solely on response propensity.

Methods up until now are generally referred to as randomization approaches: population parameters are assumed fixed and variation is introduced due to the probability distribution used to select the sample of people chosen to receive the survey form. Model-based approaches, where “the population values are treated as realisations of random variables that are distributed according to some model” (Little
1982) have also been used in an attempt to reduce nonresponse bias in mail and other survey types (e.g. Kaufman & King 1973; Albert & Gupta 1985). These methods require making assumptions about the distribution that the population parameters follow and then estimating the parameters of these distributions such that the likelihood given the observed data is maximized. The distribution along with these estimated parameters are then assumed to describe the population parameters of interest.

Stasny (1991) used a hierarchical model approach to estimate the probability that an individual in the population being surveyed had been a victim of crime using interview data that suffered from nonresponse. The sampled population was stratified into 10 domains (based on housing descriptions and income level). The hierarchical model assumed that a person responding was dependent on whether or not they had been victimized. Assuming different distributions for each of these required variables, this method estimates the parameters for each of these distributions such that the likelihood of the observed data is maximized under the hierarchical model that describes the observed sample. Using all data in the one analysis, this approach was able to produce reliable estimates for domains that had only a small number of respondents. A similar modelling approach was also used by Nandram & Choi (2002) in a national health survey.

Model based approaches rely on appropriate choices of distributions for the population parameters under study.

Jackman (1999) used auxiliary information in the form of other performed surveys, and not data directly related to the survey being performed, to reduce the range on Manski’s Bounds. This study attempted to account for both nonresponse and measurement error in the reported response to a binary attribute of a survey of Australians to ascertain whether or not they would vote, if voting was voluntary. Similar surveys with known voter turn out in New York (13 surveys) and New Zealand (2 surveys) were used to approximate a multivariate normal distribution describing the pairwise occurrence of the parameter adjustments for the aforementioned sources of error in estimating the population parameter of interest. The mode of this resulting distribution, estimated by random sampling 100 duplets
from the range of each sample by assuming them to follow a symmetric, triangular distribution (Evans et al. 1993), is then used as the required estimates for the Australian case study as well as 95% confidence intervals.

The success of any method in reducing the impact of nonresponse on a response variable is difficult to know if the true population value of that variable is not known. In an effort to understand how successful a method may be in reducing this bias, other more reliable, but generally more expensive survey methods, can be used and compared to the adjusted results of a MS. Some studies have taken this a step further by attempting to adjust the result of the cheaper but more biased survey method to that of the more reliable, but more expensive, survey by calculating a correction factor (e.g. Connelly & Brown 1995; Lyle et al. 1999). This method has the advantage of being able to correct for more than just nonresponse bias, but relies on these biases being consistent over time for the identified correction factor to be accurate in calibrating further future surveys without the need to running the more reliable (but more expensive) survey.
Chapter 3

Correcting the Total Catch Estimated by a Mail Survey for Nonresponse, Recall and Avidity Bias Using a Phone-Diary Survey

Historical comparisons show that the total catch estimated using a mail survey (MS) method for the recreational western rock lobster fishery is generally twice that estimated using a phone-diary survey (PDS) method (IFAAC 2007; Hartill et al. 2011). Each of these survey methods have their own level of bias but the PDS is generally considered to be more accurate than the MS and has been accepted by managers as the survey estimate to measure the future catch allocation between this sector and the commercial sector. To reduce costs, this Chapter identifies a correction factor that standardizes the total catch estimated from the MS to that expected from the PDS, so that the cheaper MS can be used each season whilst still satisfying the requirements of management.

A known index of lobster abundance is used to assess the reliability of the two survey methods to estimate the trend in total recreational catch of the western rock lobster fishery. It is seen that the catch rate information reported by the MS is positively correlated with lobster abundance and supports the usefulness of the MS as a data collection method for identifying the trend in the catches of this sector. Interestingly however, most likely due to increased sampling variation due to the lower sample sizes employed to minimize costs, the PDS has produced estimates of some parameters that do not fit well with the general relationships observed against rock lobster abundance.

To identify the correction factor, average responses of various parameters between survey methods are compared and it is observed that the probability of returning the MS form for people who fished \( (\pi_f) \) was greater than those that did not \( (\pi_{\bar{f}}) \) i.e. \( \pi_f > \pi_{\bar{f}} \). It was also seen that the probability of fishers returning the survey form
increased with their reported number of days fished (avidity bias). This higher representation of fishers and of those that fished more frequently, largely explains why the MS has estimated total catch to be much higher than the PDS.

This Chapter continues by briefly describing the recreational western rock lobster fishery and the two survey methods that have been used to estimate it’s total catch. The differences between the total catch estimates of the two survey methods are quantified and their drivers investigated. A correction factor is then derived that standardizes the total catch estimate of the MS to that expected from a PDS.

3.1 Background

Western rock lobsters (*Panulirus cygnus*) are found on the lower west coast of Western Australia (Figure 3.1), supporting the largest single species commercial fishery in Australia and an important recreational fishery. In the last decade, seasonal commercial catches have ranged from about 5 500 to 14 000 t, worth approximately A$200-400 million per annum, and more than 40 000 people have been licensed in any one season (Figure 3.2), to participate in the 7.5 month season (15 November – 30 June).

Covering other various recreational fishing activities, a person was once required to hold an amateur fishing licence to fish recreationally for rock lobster. In 1986 a series of licences specific to each of these activities were introduced to replace the amateur licence and included a licence to fish specifically for rock lobster (“RL” licence). An “umbrella” licence (“U” licence) was introduced in 1992 that, at a significant financial saving to purchasing all separately, legally allowed the licence holder to fish rock lobster and four other licensed recreational fisheries (abalone, marron, freshwater and netting). Prior to the introduction of the umbrella licence, licences would expire at the end of the financial year (30 June) in which they were purchased. The introduction of the umbrella licence saw all of these licences expire 12 months from the date of purchase. The umbrella licence was removed from sale in March of 2010.
Figure 3.1: Western rock lobsters are found along the West Australian coast between latitudes 21°44′S and 34°24′S. The current sites for puerulus collector sites (de Lestang et al. 2011) are highlighted in red. Recreational fishing generally occurs within a few kilometers (to water depths < 40m) off the main coastline of Western Australia between Kalbarri and Augusta, with most of this effort occurring off the metropolitan coastal areas of Perth.
Figure 3.2: The number of people licensed to fish for rock lobster each season by licence type. Umbrella licences (U), which allow people to fish for rock lobster and species in four other fisheries, were introduced at the end of the 1992/93 season but were removed from sale in March 2010 (those umbrella licence numbers active in 2010/11 represent those purchased prior to being removed from sale). Rock lobster specific licences (RL) have existed since the beginning of the 1986/87 season.

 Whilst the commercial sector fish solely for rock lobster using pots that comply with various management regulations, recreationally fishers also dive for lobsters (by either free, scuba or hookah diving methods). Other methods for fishing recreationally for lobster include reef walking with a torch at night but such methods are infrequent and account for < 1 tonne of the total recreational catch in any one season (Melville-Smith et al. 2000). Only catch and effort for potting and diving are considered in this research and are treated separately since the catchability of lobsters are likely to differ between these fishing methods.

 Commercial catches of western rock lobster along the coast are reliably explained by puerulus settlement 3 and 4 years earlier (Morgan et al. 1982; Phillips 1986; Caputi et al. 1995). Melville-Smith et al. (2001) and (2004) has shown that puerulus settlement at Alkimos (Figure 3.1), a location just north of the Perth metropolitan area where the majority of recreational fishing occurs, is well correlated with the recreational lobster catches 3 to 4 years later.
The predictability of rock lobster catches has allowed management to be proactive in implementing stock protective strategies using input control techniques on both the commercial and recreational sectors over time. As part of this, the western rock lobster fishery has adopted an integrated fisheries management (IFM) approach and determined fixed allocations of the total sustainable catch for a season to each of the sectors (IFAAC 2007; Crowe et al. 2013). With the catch share of the commercial and recreational sectors being set at 95% and 5% respectively, and given the range of historical catches, this could result in the recreational fishery being allocated between 250 and 700 t in a particular season.

The move to IFM requires that the catch for each sector is accurately known so that management adjustments can be made to regulate catch, if required, to maintain these proportions. The total catch of the commercial sector is determined from mandatory records of catches as part of the licensing agreement and processor returns are also available to validate the catch. Recreational fishers however, are not legally required to report their catches and so an end of season MS has been used to estimate the total catch of this fishery since 1986/87.

The use of MSs is cost effective for collecting information required for estimating the total catch of recreational fishers (Fox et al. 1998). A number of studies have raised concerns however, about low response rates potentially adding a nonresponse bias to these estimates (Brown 1991; Page 1991; Strayer et al. 1993; Stein et al. 1999). Another concern with MSs is that this survey method typically questions people about fishing activities that have already occurred (up to many months earlier) and consequently, the MS may suffer from recall bias (Connelly & Brown 2011). A study by van Kenhove et al. (2002) also suggests that MSs may suffer from an avidity bias where “on average high involvement” subjects are more likely to return a MS form than those less involved. In the case of a MS of recreational anglers, this might suggest that more frequent fishers are more likely to return a survey form than those less frequent fishers.

PDSs are a more expensive but potentially more accurate method than MSs for estimating the total catch of recreational fisheries (McGlennon 1999; Lyle et al.
A PDS is generally considered to be less affected by nonresponse, recall and avidity bias than MSs because all survey participants are invited to be part of the survey before the fishing has occurred and are encouraged to accurately record the outcomes of this activity throughout the season by direct regular contact with an interviewer who collects this information.

To determine if MS estimates of total catch could be used by managers to reliably assess the total catch of the western rock lobster recreational fishery, it was decided to also run a much more expensive, but much considered more reliable, PDS method for several seasons (2000/01, 2001/02, 2004/05 – 2008/09). Over the period in which both survey methods have been used, the return rates of survey forms for the MSs have ranged between 41 – 51% while the refusal rates in the screening surveys to enrol diarists to the PDSs were much lower (≤ 10%) (Table 3.1).

Due to the lower refusal rates and the general likelihood to produce more reliable estimates than the MS, managers and stakeholders have accepted the PDS survey as the standard method to assess whether the total catch of the western rock lobster recreational fishery is within its allocated 5% catch share allocation. Rather than run this more expensive survey method every season, it is hoped that a reliable correction factor that adjusts the total catch estimate derived from a MS to that from a PDS can be identified, so that the MS can be run each year and then simply adjusted to what would be expected from the PDS.

### 3.2 Total Catch Estimation for Each Survey Method

Both rock lobster specific and umbrella licence holders provide a reference frame for identifying people that can legally fish recreationally for rock lobster and are the primary sampling units for the MS and PDS. Whilst there are major differences in how they are conducted, both survey methods collect information that allow the total catch of the recreational western rock lobster to be calculated in terms of its primary fishing methods of potting and diving. A summary of the information collected by the MS and PDS to derive these estimates is presented in Table 3.2.
Table 3.1: The initial and net sample size (the number of people initially sampled less those that could not be contacted e.g. address unknown, disconnected number, business number and language issues) as well as the number and percentage of these contacted that responded (Resp.) or agreed to participate in the survey, for seasons that both the mail and phone-diary screening surveys were run. The size of the surveyed population for each survey method (Popu.), and the net sampling fraction of the population, is also presented.

<table>
<thead>
<tr>
<th>Season</th>
<th>Mail survey</th>
<th></th>
<th>Phone-diary screening survey</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2000/01</td>
<td>40807</td>
<td>4000</td>
<td>3943</td>
<td>10%</td>
</tr>
<tr>
<td>2001/02</td>
<td>40714</td>
<td>4000</td>
<td>3941</td>
<td>10%</td>
</tr>
<tr>
<td>2004/05</td>
<td>44643</td>
<td>4000</td>
<td>3909</td>
<td>9%</td>
</tr>
<tr>
<td>2005/06</td>
<td>41563</td>
<td>6000</td>
<td>5793</td>
<td>14%</td>
</tr>
<tr>
<td>2006/07</td>
<td>41178</td>
<td>4000</td>
<td>3963</td>
<td>10%</td>
</tr>
<tr>
<td>2007/08</td>
<td>40452</td>
<td>4000</td>
<td>3936</td>
<td>10%</td>
</tr>
<tr>
<td>2008/09</td>
<td>41917</td>
<td>3000</td>
<td>2993</td>
<td>7%</td>
</tr>
</tbody>
</table>
Table 3.2: Summary of the data collected from the mail (MS) and phone-diary survey (PDS), required to estimate total retained catch for potting and diving, for each season that both surveys were performed for the recreational western rock lobster fishery. The population size ($N_{t,s}$), proportion of respondents reporting having potted ($\hat{p}_{t,\text{pot}}^{(s)}$) and dived ($\hat{p}_{t,\text{dive}}^{(s)}$), and the average retained catch per fisher by potting ($\hat{F}_{t,\text{pot}}^{(s)}$) and diving ($\hat{F}_{t,\text{dive}}^{(s)}$), are presented for survey method $s$ and season $t$.

<table>
<thead>
<tr>
<th>Season ($t$)</th>
<th>$N_{t,s}$</th>
<th>$\hat{p}_{t,\text{pot}}^{(s)}$</th>
<th>$\hat{p}_{t,\text{dive}}^{(s)}$</th>
<th>$\hat{F}_{t,\text{pot}}^{(s)}$</th>
<th>$\hat{F}_{t,\text{dive}}^{(s)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000/01</td>
<td>40807</td>
<td>0.54</td>
<td>0.32</td>
<td>42.0</td>
<td>25.2</td>
</tr>
<tr>
<td>2001/02</td>
<td>40714</td>
<td>0.46</td>
<td>0.30</td>
<td>42.4</td>
<td>28.3</td>
</tr>
<tr>
<td>2004/05</td>
<td>44643</td>
<td>0.50</td>
<td>0.27</td>
<td>48.7</td>
<td>32.8</td>
</tr>
<tr>
<td>2005/06</td>
<td>41563</td>
<td>0.43</td>
<td>0.26</td>
<td>33.6</td>
<td>18.9</td>
</tr>
<tr>
<td>2006/07</td>
<td>41178</td>
<td>0.38</td>
<td>0.26</td>
<td>31.7</td>
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<td>2007/08</td>
<td>40452</td>
<td>0.39</td>
<td>0.28</td>
<td>36.5</td>
<td>19.9</td>
</tr>
<tr>
<td>2008/09</td>
<td>41917</td>
<td>0.36</td>
<td>0.29</td>
<td>34.2</td>
<td>25.2</td>
</tr>
</tbody>
</table>

Mail survey method

A MS has been carried out for each western rock lobster fishing season since 1986/87. At the end of each season, generally 4000 people (or, approximately 10%) that were licensed to fish for rock lobster during that season were randomly selected (independent of licence type) and mailed a survey form (see Table 5.2 in Chapter 5 for a comprehensive account of the number of surveys sent each season relative to the number of people licensed to fish for rock lobster). Due to the change in licence expiry date from “end of season” to “12 months after purchase” at the start of the 1992/93 season, the random licence extraction for that and subsequent seasons included any person who purchased their licence within 18 months before the end of the season being surveyed. In this way, everyone who could have fished any part of that season had the chance of being selected to receive a survey form. Whilst the survey form has evolved to include additional questions of varying detail over time (see Appendix A & B for examples of the survey forms used), the form has always surveyed the number of days fished, and how many lobsters were retained, by the different fishing methods (potting, diving or ‘other’) at a whole-of-season level.
Under the MS method, the number of people who could have fished for rock lobster in season $t$ is denoted by $N_{t}^{(m)}$. The returned survey forms are then used to make the following parameter estimates for each fishing method:

- $\widehat{p}_{t,v,o}^{(m)}$ - the proportion of licensees that fished for lobster using fishing method $v$ estimated using the ratio estimator ($\widehat{\gamma}_{o}$) i.e. $\widehat{\gamma}_{o} = n_{x}/(n_{x} + n_{\bar{x}})$ where $n_{x}$ and $n_{\bar{x}}$ is the number of survey returns that reported having characteristic $x$ or not, respectively;
- $\hat{L}_{t,v,o}^{(m)} = N_{t}^{(m)} p_{t,v,o}^{(m)}$ - the number of licensees that fished using method $v$ as estimated by $\widehat{\gamma}_{o}$, and
- $\hat{F}_{t,v}^{(m)}$ - the average retained catch per fisher (in number of lobsters) reported by using fishing method $v$, given they fished using fishing method $v$.

The total catch estimate for fishing method $v$ in season $t$ using the MS method ($\widehat{\gamma}_{t,v}$) is then defined as:

$$\widehat{\gamma}_{t,v}^{(m)} = \hat{L}_{t,v,o}^{(m)} \hat{F}_{t,v}^{(m)}$$

The catch estimates in numbers are converted to kilograms by applying an assumed average weight of 0.5 kg per lobster (Melville-Smith et al. 2004).

**Phone-diary survey (PDS) method**

The PDS design as described by Baharthah (2007), which is similar to that used by Fishcount in the Northern Territory (Coleman, 1998; Lyle et al., 2002), has been carried out in seven seasons (2000/01, 2001/02, 2004/05 – 2008/09). An initial screening phase is carried out approximately 6 weeks prior to the start of each season, where randomly selected people who are licensed to fish recreationally for rock lobster at that time are contacted and asked to participate in the survey for the upcoming season. The enrolment in the subsequent diary phase continues until a quota of diarists is achieved and has varied between 400 and 800 (Table 3.1) with half being rock lobster specific licence holders and the other, umbrella licence holders.
The successfully screened respondents are sent a diary in which they can record their
daily rock lobster fishing activity (when, where, method of fishing and how many
lobsters they retained and released). Diarists are phoned each month and an
interviewer records the details of their fishing activity for that period. Whenever
possible, the same interviewer is maintained for each diarist throughout the survey to
encourage continued participation and reporting of accurate data by attempting to
develop rapport between diarist and interviewer.

At the end of each season, in a similar way to the MS, total catch estimates are then
calculated for both potting and diving with the exception of two differences. Firstly,
the number of people with licence type \(k\) in a particular season is assumed to be the
average number of people with that licence type on the last day of each month of that
season \(N_i^{(pds)k}\). Secondly, the total catch for each fishing method is calculated for
each licence type and then added together since whilst RL and U licence holders
occur equally in the survey sample, the number of people who hold these licences do
not (Figure 3.2).

Under the PDS, the number of people estimated to hold licence type
\(k \in \{R = RL, U = Umbrella\}\) in season \(t\) is denoted by \(N_i^{(pds)k}\). The responses of
PDS participants are then used to estimate the following parameters for each fishing
method \(v \in \{pot, dive\}\) in season \(t\):

- \(\hat{\gamma}_i^{(pds)k}\) - the proportion of people with licence type \(k\) that fished for lobsters
  using fishing method \(v\), estimated using \(\hat{\gamma}_i\);
- \(\hat{\bar{L}}_{t,v,o}^{(pds)k} = N_t^{(pds)k} \cdot \hat{p}_{t,v,o}^{(pds)k}\) - the number of people with licence type \(k\) estimated
to have fished using method \(v\); and
- \(\hat{\bar{F}}_{t,v}^{(pds)k}\) - the average number of lobsters caught and retained by people with
  licence type \(k\) using fishing method \(v\), given they fished using fishing method
  \(v\).
Chapter 3

The total catch estimate for fishing method $v$ in season $t$ using the PDS method ($\hat{T}_{t,v}^{(pds)}$) is then defined as:

$$\hat{T}_{t,v}^{(pds)} = \hat{T}_{t,v}^{(pds)R} \hat{R}_{t,v}^{(pds)R} + \hat{T}_{t,v}^{(pds)U} \hat{U}_{t,v}^{(pds)U}$$

As with the MS, the catch estimates in numbers are converted to kilograms by applying an assumed average weight of 0.5 kg per lobster.

Independent of licence type, the number of fishers and average retained catch per fisher estimated by the PDS for fishing method $v$ in season $t$ are defined respectively as:

$$\hat{F}_{t,v}^{(pds)} = \hat{F}_{t,v}^{(pds)R} + \hat{F}_{t,v}^{(pds)U} \quad \text{and} \quad \hat{F}_{t,v} = \frac{\hat{F}_{t,v}^{(pds)}}{\hat{L}_{t,v}^{(pds)}}.$$

### 3.3 Comparison of Key Parameter Estimates between Survey Methods

To reliably use a correction factor to standardize the MS catch estimates to those of the PDS, it is necessary for differences between the parameters comprising those estimates, to be consistent between seasons. To aid comparison of the different parameters, their 95% confidence intervals are presented to indicate whether differences between the two survey methods are statistically significant. The confidence intervals for all parameters are generated using the percentile bootstrap method with 1000 simulations.

Umbrella and rock lobster specific licence holders are readily identified in the PDS but not the MS. Comparing $\hat{F}_{t,v}^{(pds)U}$ and $\hat{F}_{t,v}^{(pds)RL}$ each season for potters (Figure 3.3) and divers (Figure 3.4), it is seen that these estimates are statistically similar with no obvious bias. Similarly comparing $\hat{P}_{t,v}^{(pds)U}$ and $\hat{P}_{t,v}^{(pds)RL}$, it is seen that there was no significant difference between licence types for diving (Figure 3.5) but there is for potting (Figure 3.6). Given the observed difference between $\hat{P}_{t,v}^{(pds)U}$ and $\hat{P}_{t,v}^{(pds)RL}$ for
potting is only marginal, it will be assumed that any significant differences in the
total catch estimates between the two survey methods for this source of bias is
minimal.

Comparing \( \hat{T}_{t,v,o}^{(\text{pds})} \) and \( \hat{T}_{t,v,o}^{(\text{m})} \) for each fishing method \( v \in \{ \text{pot, dive} \} \) it is generally
seen that \( \hat{T}_{t,v,o}^{(\text{m})} \) is approximately 150 – 200\% higher than \( \hat{T}_{t,v,o}^{(\text{pds})} \) each season \( t \) (Table
3.3). Whilst \( \hat{L}_{t,v,o}^{(s)} \), \( s \in \{ \text{Mail, pds = PDS} \} \), is seen to explain a significant
proportion of this error (\( \hat{L}_{t,v,o}^{(\text{m})} \) is generally 40 - 60\% greater than \( \hat{L}_{t,v,o}^{(\text{pds})} \), Table 3.3), \( \hat{F}_{t,v}^{(s)} \) is the biggest contributor to this difference (\( \hat{F}_{t,v}^{(\text{m})} \) generally 70 - 100\% greater
than \( \hat{F}_{t,v}^{(\text{pds})} \), Table 3.3). The differences between \( \hat{T}_{t,v,o}^{(s)} \) and \( \hat{T}_{t,v}^{(s)} \) for each survey and
fishing method are observed in Figures 3.7 – 3.10. It is seen that the 95\% confidence
intervals for each of the estimated parameters between survey methods are non-
overlapping and hence, statistically different. Even more, it is seen that the estimates
for each of these parameters using the PDS are consistent lower than for the MS.

![Figure 3.3: The average retained catch per fisher for potters as estimated by the
phone-diary survey for both umbrella and rock lobster specific licence holders. 95\%
percentile bootstrap confidence intervals constructed using 1000 simulations are
included for each estimate.](image-url)
Figure 3.4: The average retained catch per fisher for divers as estimated by the phone-diary survey for both umbrella and rock lobster specific licence holders. 95% percentile bootstrap confidence intervals constructed using 1000 simulations are included for each estimate.

Figure 3.5: Proportion of licensees who reported diving as estimated by the phone-diary survey for both umbrella and rock lobster specific licence holders. 95% percentile bootstrap confidence intervals constructed using 1000 simulations are included for each estimate.
Figure 3.6: Proportion of licensees who reported potting as estimated by the phone-diary survey for both umbrella and rock lobster specific licence holders. 95% percentile bootstrap confidence intervals constructed using 1000 simulations are included for each estimate.

Although \( \hat{F}^{(m)}_{t,v} \) is generally 70 - 100% greater than \( \hat{F}^{(pds)}_{t,v} \) over time (Table 3.3), it is evident in Figures 3.9 and 3.10 that \( \hat{F}^{(m)}_{t,v} \) are correlated with the puerulus settlement at Alkimos 3 and 4 years prior, which is an index of lobster abundance, for both potting and diving. Plotting \( \hat{F}^{(x)}_{t,v} \) for each survey and fishing method against puerulus settlement, Figures 3.11 & 3.12 illustrate the significance of this correlation. The catch per unit effort (total catch divided by total days fished) is also compared (Figures 3.13 & 3.14) against puerulus settlement. The significant correlation illustrated in these graphics support the usefulness of both survey methods to estimate the trend of total catch since their seasonal variation reflects the variation in abundance of lobsters each season. It is noted however, that the correlations between \( \hat{F}^{(pds)}_{t,v} \) and puerulus settlement are not as strong as with \( \hat{F}^{(m)}_{t,v} \) for both fishing methods even when removing from consideration the possible outlier for the PDS.
Table 3.3: The relative percentage increase of the mail survey (MS) total catch estimate ($\hat{\theta}_{t,v,o}^{(m)}$) compared to that of the corresponding phone-diary survey (PDS) ($\hat{\theta}_{t,v,o}^{(pds)}$), for each fishing method $v$ and season $t$ when both survey methods were used for the recreational western rock lobster fishery. The relative percentage difference between the two survey methods is also presented for the estimated number of fishers ($\hat{N}_{t,v,o}$) and average retained catch per fisher ($\hat{L}_{t,v,o}$) are also presented i.e. $\text{RelI} = 100\frac{y^{(m)} - y^{(pds)}}{y^{(pds)}}$. The number of respondents who reported fishing using method $v$ for the MS ($n_{t,v}^{(m)}$) and PDS ($n_{t,v}^{(pds)}$) are also reported.

<table>
<thead>
<tr>
<th>Fishing Method $(v)$</th>
<th>Season $(t)$</th>
<th>Respondents</th>
<th>RelI = $100\frac{y^{(m)} - y^{(pds)}}{y^{(pds)}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$n_{t,v}^{(m)}$</td>
<td>$n_{t,v}^{(pds)}$</td>
</tr>
<tr>
<td>Potting</td>
<td>2000/01</td>
<td>1075</td>
<td>194</td>
</tr>
<tr>
<td></td>
<td>2001/02</td>
<td>831</td>
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<td></td>
<td>2004/05</td>
<td>833</td>
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</tr>
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<td>2005/06</td>
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</tr>
<tr>
<td></td>
<td>2008/09</td>
<td>434</td>
<td>106</td>
</tr>
<tr>
<td>Diving</td>
<td>2000/01</td>
<td>635</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>2001/02</td>
<td>533</td>
<td>108</td>
</tr>
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<td>2004/05</td>
<td>454</td>
<td>97</td>
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<td></td>
<td>2005/06</td>
<td>630</td>
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<td></td>
<td>2006/07</td>
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<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>2008/09</td>
<td>345</td>
<td>88</td>
</tr>
</tbody>
</table>
Figure 3.7: The number of people who potted for rock lobster each season as estimated by the mail and phone-diary survey along with their 95% percentile bootstrap confidence intervals estimated using 1000 simulations. The index for lobster biomass (Puerulus) as measured by the average puerulus settlement at Alkimos 3 and 4 years prior, is also presented.

Figure 3.8: The number of people who dived for rock lobster each season as estimated by the mail and phone-diary survey along with their 95% percentile bootstrap confidence intervals estimated using 1000 simulations. The index for lobster biomass (Puerulus) as measured by the average puerulus settlement at Alkimos 3 and 4 years prior, is also presented.
Figure 3.9: The average retained catch per fisher for people who potted for rock lobster each season as estimated by the mail and phone-diary survey along with their 95% percentile bootstrap confidence intervals estimated using 1000 simulations. The index for lobster biomass (Puerulus) as measured by the average puerulus settlement at Alkimos 3 and 4 years prior, is also presented.

Figure 3.10: The average retained catch per fisher for people who dived for rock lobster each season as estimated by the mail and phone-diary survey along with their 95% percentile bootstrap confidence intervals estimated using 1000 simulations. The index for lobster biomass (Puerulus) as measured by the average puerulus settlement at Alkimos 3 and 4 years prior, is also presented.
Figure 3.11: The average retained catch per fisher for potters, plotted against average puerulus settlement at Alkimos 3 and 4 years prior, using data from the (A) mail (MS) and (B) phone-diary survey (PDS). Each data set is presented with a line of best fit which, for the PDS, has been fitted with 08/09 excluded ($\hat{\rho} = 0.62$, $P = 0.19$, df = 4) since it produces a statistically better fit than when including all seasons ($\hat{\rho} = 0.22$, $P = 0.64$, df = 5). For the MS, the line of best fit was constructed using all data ($\hat{\rho} = 0.85$, $P < 0.01$, df = 23).

Figure 3.12: The average retained catch per fisher for divers, plotted against average puerulus settlement at Alkimos 3 and 4 years prior, using data from the (A) mail (MS) and (B) phone-diary survey (PDS). The line of best fit is included for both the MS ($\hat{\rho} = 0.74$, $P < 0.01$, df = 23) and PDS ($\hat{\rho} = 0.34$, $P = 0.46$, df = 5).
Figure 3.13: The estimated CPUE (average retained catch per fishing day) for potters, plotted against average puerulus settlement at Alkimos 3 and 4 years prior, using data from the (A) mail (MS) and (B) phone-diary survey (PDS). Each data set is presented with a line of best fit which for the PDS has been fitted with 08/09 excluded ($\rho = 0.94$, $P < 0.01$, df = 4) since it produces a statistically better fit than when including all seasons ($\rho = 0.55$, $P = 0.20$, df = 5). For the MS, the line of best fit was constructed using all data ($\rho = 0.71$, $P < 0.01$, df = 23).

Figure 3.14: The estimated CPUE (average retained catch per fishing day) for divers, plotted against average puerulus settlement at Alkimos 3 and 4 years prior, using data from the (A) mail (MS) and (B) phone-diary survey (PDS). The line of best fit for both the MS ($\rho = 0.61$, $P < 0.01$, df = 23) and PDS ($\rho = 0.34$, $P = 0.46$, df = 5) are also included.
With $\widehat{L}_{t,v,o}^{(m)}$ being significantly greater than $\widehat{L}_{t,v,o}^{(pds)}$ for both potting (Figure 3.7) and diving (Figure 3.8) for all seasons suggests that people who fished are more likely to return the MS form than people who did not. Explanations as to why $\widehat{F}_{t,v}^{(m)}$ and $\widehat{F}_{t,v}^{(pds)}$ differ markedly for the two fishing methods is investigated by comparing the proportion of fishers, and their average retained catch, for various ranges of reported days fished. MSs are well known to suffer from rounding errors where respondents have a tendency to report to the nearest 5 (Beaman et al. 1997). To reduce the impact of this bias on these comparisons, it was decided to compare fishers within a range of days that represent multiples of 5 e.g. 1 – 5, 6 – 10 and 11 – 15. Given that the sample sizes and range of days fished for the PDS were much smaller than for the MS (Figure 3.15), the ranges were chosen so that the PDS had a substantial sample size to make comparisons reliable. Referring to Table 3.4, it is decided to partition potters into five day ranges (1 – 5, 6 – 10, 11 – 15, 16 – 20 and >20) and divers into two (1 – 5 and >5).

Within each of the defined ranges of days fished, $\widehat{F}_{t,\text{pot}}^{(s)}$ are very similar between the two survey methods for all day ranges considered for potting (Figure 3.16) with the exception of the least avid fisherman (1 – 5 days). In this case, the MS respondents generally reported slightly higher levels of total retained catch than the PDS. Averaged over all days fished, the difference in $\widehat{F}_{t,\text{pot}}^{(s)}$ between the two survey methods is then seen to be explained by the different proportions of potters that reported having fished 1 – 5 days and those having reported fishing >20 days (Figure 3.17).

For both of the day classes considered for diving, it is seen that $\widehat{F}_{t,\text{dive}}^{(s)}$ is similar between survey methods (Figure 3.18) but that the avid fisherman (>5 days) were more likely to return the MS form than the less avid (1 – 5 days) (Figure 3.19).
Figure 3.15: Boxplot of the days fished each season as reported by respondents to the mail (MS) and phone-diary survey (PDS) grouped by fishing method (potting or diving). The range of each boxplot marks the 2.5%, 25%, 50%, 75% and 97.5% percentiles of the dataset that it corresponds i.e. outliers are excluded.

Table 3.4: The number of phone-diary survey participants that reported fishing a range of days by either potting or diving for each season that this survey type was run.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Potting</td>
<td>1-5</td>
<td></td>
<td>54</td>
<td>42</td>
<td>71</td>
<td>50</td>
<td>87</td>
<td>77</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td></td>
<td>41</td>
<td>38</td>
<td>35</td>
<td>30</td>
<td>42</td>
<td>41</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>11-15</td>
<td></td>
<td>34</td>
<td>16</td>
<td>10</td>
<td>9</td>
<td>28</td>
<td>21</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>16-20</td>
<td></td>
<td>12</td>
<td>19</td>
<td>6</td>
<td>5</td>
<td>12</td>
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<tr>
<td></td>
<td>&gt;25</td>
<td></td>
<td>43</td>
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<td>123</td>
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<td>16</td>
<td>8</td>
<td>5</td>
<td>12</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>
Figure 3.16: The average retained catch per fisher by potting for fishers that reported potting (A) 1-5 days; (B) 6-10 days; (C) 11-15 days; (D) 16-20 days; and (E) >20 days, as estimated by the mail (green) and phone-diary survey (black) for each season. 95% percentile bootstrap confidence intervals using 1000 simulations are also included for all estimates.
Figure 3.17: The proportion of people who reported potting that reported only potting (A) 1–5 days; (B) 6–10 days; (C) 11–15 days; (D) 16–20 days; and (E) >20 days, as estimated by the mail (green) and phone-diary survey (black) for each season. 95% percentile bootstrap confidence intervals using 1000 simulations are also included for all estimates.
Figure 3.18: The average retained catch per fisher by diving for fishers that reported diving (A) 1-5 days; and (B) >5 days, as estimated by the mail (green) and phone-diary survey (black) for each season. 95\% percentile bootstrap confidence intervals using 1000 simulations are also included for all estimates.
Figure 3.19: Proportion of people who reported diving, that reported only diving (A) 1-5 days; and (B) >5 days, as estimated by the mail (green) and phone-diary survey (black) for each season. 95% percentile bootstrap confidence intervals using 1000 simulations are also included for all estimates.
3.4 Calculating a Correction Factor

Summarizing to here, \( \widehat{T}_{t,v,o}^{(m)} \) being significantly larger than \( \widehat{T}_{t,v,o}^{(pds)} \) for each season \( t \) and fishing method \( v \) is largely explained by:

1. \( \widehat{L}_{t,v,o}^{(m)} \) being significantly greater than \( \widehat{L}_{t,v,o}^{(pds)} \) due to fishers being more likely to return a MS form; and

2. \( \widehat{F}_{t,v}^{(m)} \) being significantly greater than \( \widehat{F}_{t,v}^{(pds)} \) due to fishers who fished more days, and hence tending to catch more, being more likely to return the MS form.

The consistent differences in \( \widehat{L}_{t,v,o}^{(s)} \) and \( \widehat{F}_{t,v}^{(s)} \) between the two survey methods (Table 3.3; Figures 3.20 & 3.21) indicates that a correction factor can be applied to each of these parameters that will result in reducing the scale of \( \widehat{T}_{t,v,o}^{(m)} \) to that of \( \widehat{T}_{t,v,o}^{(pds)} \).

To determine the correction factor to standardize \( \widehat{T}_{t,v,o}^{(m)} \) to the level of \( \widehat{T}_{t,v,o}^{(pds)} \), the input parameters from the PDS are modelled in terms of those of the corresponding MS estimate:

\[
\widehat{F}_{t,v}^{(pds)} = a_F + b_F \widehat{F}_{t,v}^{(m)} + \varepsilon_{F,t}
\]

and

\[
\widehat{L}_{t,v,o}^{(pds)} = a_L + b_L \widehat{L}_{t,v,o}^{(m)} + \varepsilon_{L,t}
\]

where \( \varepsilon_{L,t} \sim N(0, \sigma_L^2) \), \( \varepsilon_{F,t} \sim N(0, \sigma_F^2) \) and \( v \in \{pot, dive\} \).

Least-squares regressions between the PDS and MS estimates of \( \widehat{T}_{t,v,o}^{(s)} \) (Figure 5.20) and \( \widehat{F}_{t,v}^{(s)} \) (Figure 5.21) are presented in Table 3.5. The intercepts for each model describing \( \widehat{L}_{t,v,o}^{(pds)} \) and \( \widehat{F}_{t,v}^{(pds)} \), in terms of \( \widehat{T}_{t,v,o}^{(m)} \) and \( \widehat{F}_{t,v}^{(m)} \) respectively, are nonsignificant (Table 3.5, \( P > 0.68 \)) and hence, intercepts are not included in the optimal models (model II for both \( \widehat{L}_{t,v,o}^{(s)} \) and \( \widehat{F}_{t,v}^{(s)} \), Table 3.5).
In fitting the regression for $\hat{F}_{t,v}^{(pds)}$ it is seen that there exist suspected outliers (Figure 3.21). Since however the parameter estimates of the optimal model for $\hat{F}_{t,v}^{(pds)}$ fitted using all data (Model III: 0.5653; Table 3.5) is similar to that fitted excluding data of the outliers (Model II: 0.5938; Table 3.5), all data is used in determining the correction factor for $\hat{F}_{t,v}^{(m)}$.

Using the no-intercept models fitted using all data, and assuming that the gradients $b_L$ and $b_F$ are independent, the correction factor of 0.39 ($b_L \times b_F = 0.6576 \times 0.5938$, Table 3.5) is identified to standardize $\hat{F}_{t,v,o}^{(m)}$ to $\hat{F}_{t,v,o}^{(pds)}$. The standard error for this correction factor can be approximated using (Kendall & Stuart, 1963):

$$
\sigma_{\hat{F}}^2 \approx (\hat{x} \times \hat{y})^2 \left( \frac{\sigma_x^2}{\hat{x}^2} + \frac{\sigma_y^2}{\hat{y}^2} + 2 \frac{\sigma_{xy}}{\hat{x}\hat{y}} \right)
$$

Since $b_L$ and $b_F$ have been assumed independent, $\sigma_{\hat{F}}$ is set to zero(0) and the standard error of the correction factor is estimated as 0.0324 ($\sqrt{(0.6576 \times 0.5938)^2(0.0142^2 + 0.0473^2)}$, inputs taken from Table 3.5).

The validity of assuming $b_L$ and $b_F$ are independent is equivalent to the point estimates $\hat{L}_{t,v,o}^{(pds)} / \hat{L}_{t,v,o}^{(mail)}$ and $\hat{F}_{t,v}^{(pds)} / \hat{F}_{t,v}^{(mail)}$ being independent. This follows from the fact that, since no intercepts are fitted, $\hat{L}_{t,v,o}^{(pds)} = b_L \hat{L}_{t,v,o}^{(mail)}$ means that an estimate of $b_L$ can be approximated by $\hat{b}_L = \hat{L}_{t,v,o}^{(pds)} / \hat{L}_{t,v,o}^{(mail)}$ for each season that both survey methods were used. The same is true for $b_F$. Referring to Figure 3.22, it is seen that there is a nonsignificant correlation between $\hat{L}_{t,v,o}^{(pds)} / \hat{L}_{t,v,o}^{(mail)}$ and $\hat{F}_{t,v}^{(pds)} / \hat{F}_{t,v}^{(mail)}$ (Figure 3.22, $P = 0.79$) and hence, $b_L$ and $b_F$ being independent is a valid assumption.

The estimates $\hat{F}_{t,v,o}^{(m)}$ corrected by the factor of 0.39 (s.e. = 0.0324) ($\hat{F}_{t,v,o}^{(m)*}$), are presented in Figures 3.23 – 3.25, along with their 95% percentile bootstrapped confidence intervals using 1000 simulations. The estimates $\hat{F}_{t,v,o}^{(m)}$ and $\hat{F}_{t,v,o}^{(pds)}$ are also presented for comparison.
Previously observed in Figures 3.17 & 3.19, an important difference between the MS and PDS is the distribution of reported days fished for each fishing method. Restricting attention to seasons when both the MS and PDS were run, and using the proportion of “frequent” fishers to describe the distribution of reported days potted (fished > 10 days) and dived (fished >5 days) estimated by the PDS each season, Figure 3.26 highlights the accuracy of the single correction factor identified in this study to accurately adjust \( \hat{T}_{t,v,0}^{(m)} \) to \( \hat{T}_{t,v,0}^{(pds)} \). This accuracy is dependent on the similarity between these distributions each season for potting (Figure 3.26 A, \( P < 0.01 \)) but not diving (Figure 3.26 B, \( P = 0.46 \)).

Figure 3.20: Plot of the phone-diary survey estimates of number of fishers (’000), for each fishing method potting and diving, versus those estimated by the mail survey each season. The regression line with an intercept fitted is also presented.
Figure 3.21: Plot of the phone-diary survey estimates of average retained catch per fisher (numbers of lobsters) for each fishing method potting and diving, versus those estimated by the mail survey each season. The regression line with an intercept fitted is also presented. Suspected outliers are highlighted by the season for which they correspond.

Table 3.5: Summary of the least-squares regressions describing the phone-diary survey estimates of number of fishers ($\hat{L}_{t,v,o}^{(pds)}$) and average retained catch per fisher ($\hat{F}_{t,v}^{(pds)}$), in terms those estimated by the corresponding mail survey, independent of fishing method $v \in \{\text{pot, dive}\}$. Models with (I) and without an intercept (II) are presented as well as various commonly used statistics useful for comparing these models. For $\hat{F}_{t,v}^{(pds)}$, a model III is also included that is equivalent to model II but for the removal of data for seasons 2000/01, 2004/05 and 2008/09 for potting and 2004/05 for diving which were highlighted as possible outliers in Figure 3.21.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>R²</th>
<th>Parameter</th>
<th>value</th>
<th>s.e.</th>
<th>df</th>
<th>t-value</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{L}_{t,v,o}^{(s)}$</td>
<td>I</td>
<td>0.92</td>
<td>$a_L$</td>
<td>-380.8</td>
<td>915.9</td>
<td>12</td>
<td>-0.42</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_L$</td>
<td>0.6815</td>
<td>0.0595</td>
<td>12</td>
<td>11.46</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>-</td>
<td>$b_L$</td>
<td>0.6576</td>
<td>0.0142</td>
<td>13</td>
<td>46.12</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>$\hat{F}_{t,v}^{(s)}$</td>
<td>I</td>
<td>0.47</td>
<td>$a_F$</td>
<td>-0.83</td>
<td>6.16</td>
<td>12</td>
<td>-0.14</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_F$</td>
<td>0.6184</td>
<td>0.1883</td>
<td>12</td>
<td>3.28</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>-</td>
<td>$b_F$</td>
<td>0.5938</td>
<td>0.0475</td>
<td>13</td>
<td>12.51</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>-</td>
<td>$b_F$</td>
<td>0.5653</td>
<td>0.0092</td>
<td>9</td>
<td>61.27</td>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>
Figure 3.22: The ratio of number of fishers \( \frac{\bar{L}_{t,v,o}^{(pds)}}{\bar{L}_{t,v,o}^{(m)}} \) versus the ratio of average retained catch per fisher \( \frac{\bar{F}_{t,v}^{(pds)}}{\bar{F}_{t,v}^{(m)}} \) estimated by the phone-diary and mail survey methods, for \( v \in \{ \text{pot, dive} \} \) and each season \( t \). The correlation between \( \frac{\bar{L}_{t,v,o}^{(pds)}}{\bar{L}_{t,v,o}^{(m)}} \) and \( \frac{\bar{F}_{t,v}^{(pds)}}{\bar{F}_{t,v}^{(m)}} \) is 0.08 (\( P = 0.79 \)).
Figure 3.23: Estimated potting total catch in tonnes, with 95% percentile bootstrap confidence intervals constructed using 1000 simulations, as estimated by the raw mail survey ($\hat{T}_{t,\text{pot,o}}^{(m)}$), the raw phone-diary survey ($\hat{T}_{t,\text{pot,o}}^{(pds)}$) and the mail survey estimate adjusted to that expected from the phone-diary survey ($\hat{T}_{t,\text{pot,o}}^{(m)} = 0.39\hat{T}_{t,\text{pot,o}}^{(m)}$). Variation due to the estimated correction factor for calculating $\hat{T}_{t,\text{pot,o}}^{(m)}$ is included in the confidence interval of this estimate by recalculating its value using the randomly drawn data used at each stage of the simulation.
Figure 3.24: Estimated diving total catch in tonnes, with 95% percentile bootstrap confidence intervals constructed using 1000 simulations, as estimated by the raw mail survey ($\hat{T}_{t,\text{dive, o}}^{(m)}$), the raw phone-diary survey ($\hat{T}_{t,\text{dive, o}}^{(pds)}$) and the mail survey estimate adjusted to that expected from the phone-diary survey ($\hat{T}_{t,\text{dive, o}}^{(m)*} = 0.39\hat{T}_{t,\text{dive, o}}^{(m)}$). Variation due to the estimated correction factor for calculating $\hat{T}_{t,\text{dive, o}}^{(m)*}$ is included in the confidence interval of this estimate by recalculating its value using the randomly drawn data used at each stage of the simulation.
Figure 3.25: Estimated total catch (diving + potting) in tonnes, with 95% percentile bootstrap confidence intervals constructed using 1000 simulations, as estimated by the raw mail survey ($\hat{T}_{t,\text{pot+dive,0}}^{(m)}$), the raw phone-diary survey ($\hat{T}_{t,\text{pot+dive,0}}^{(pds)}$) and the mail survey estimate adjusted to that expected from the phone-diary survey ($\hat{T}_{t,\text{pot+dive,0}}^{(m)*} = 0.39\hat{T}_{t,\text{pot+dive,0}}^{(m)}$). Variation due to the estimated correction factor for calculating $\hat{T}_{t,\text{pot+dive,0}}^{(m)*}$ is included in the confidence interval of this estimate by recalculating its value using the randomly drawn data used at each stage of the simulation.
Figure 3.26: The proportion of (A) potters and (B) divers that reported fishing more than 10 and 5 days respectively in the phone-diary survey, versus the relative percentage difference of that fishing methods adjusted mail survey estimate of total catch $0.39\hat{T}_{t,v,o}^{(m)}$ compared to that using the phone-diary survey $\hat{T}_{t,v,o}^{(pds)}$, for each season that both survey methods were run i.e. $100(\hat{T}_{t,v,o}^{(m)} - \hat{T}_{t,v,o}^{(pds)})/\hat{T}_{t,v,o}^{(pds)}$. The lines of best fit between the variables for both potting ($\hat{\rho} = -0.96$, $P < 0.01$, df = 5) and diving ($\hat{\rho} = -0.34$, $P = 0.46$, df = 5) are also presented (dashed line).

### 3.5 Discussion

The response rate for the PDS was substantial higher than for the MS. This is typical of surveys where considerable effort is applied to personally recruit participants and then reduce the burden on them through frequent contact (Lyle et al. 2002; Hartill et al. 2011). This increased response rate however, does come at a higher cost: for the western rock lobster experience, the cost of surveying 800 people in a PDS is at least fivefold of that for a MS that samples 4000.

Along with increased response rates, the PDS leads to total catch estimates that are generally half that of the corresponding MS. A significant portion of this difference was due to $\hat{T}_{t,v,o}^{(m)}$ being substantially higher than $\hat{T}_{t,v,o}^{(pds)}$ for all seasons and fishing...
methods. Whilst the population size for the MS was slightly larger than that for the PDS, due to the difference in the way this was calculated, the difference between $\hat{L}_{t,v}^{(m)}$ and $\hat{L}_{t,v}^{(pds)}$ was primarily due to respondents in the MS being more likely to have reported fishing than those in the PDS (Table 3.2). This suggests that people who actually fished for rock lobster are more likely to return the MS form than those that did not. This result is supported by van Kenhove et al. (2002) who reported evidence of “on average high involvement” subjects being more likely to return a MS form than “on average low involvement” subjects. Nonrespondents being less active than respondents in recreational fisher surveys is also supported by Brown & Wilkins (1978). The tendency for highly involved people to return a MS form may also partly explain the higher proportion of avid fishers in the western rock lobster MS compared to that in the corresponding PDS.

The $\hat{F}_{t,\text{dot}}^{(s)}$ and $\hat{F}_{t,\text{dive}}^{(s)}$ were well correlated with the puerulus settlement index at Alkimos, an area just north of the Perth metropolitan area from which a majority of the recreational catch is taken (Melville-Smith et al. 2000), for both the MS and PDS. Being correlated with this index of lobster abundance supports the use of these survey methods to measure the trend in catches of recreational fishers. Interestingly however, the strength of these correlations were dependent on the survey and fishing method with the higher correlations being observed for the MS estimates and within the survey method, potting. The difference in correlations with lobster abundance was expected between fishing method given divers can adapt their fishing in lower catch years by increasing the size of their search area to find lobsters whilst the lobster pot is restricted to catching lobsters within the same radius of surrounding area as in higher catch years. The PDS producing estimates of $\hat{F}_{t,\text{dot}}^{(s)}$ and $\hat{F}_{t,\text{dive}}^{(s)}$ being less correlated with lobster abundance than those of the MS was likely the result of additional sampling error due to smaller sample sizes.

The number of diarists in the PDS each season was either ca. 400 (2000/01, 2001/02, 2004/05, 2005/06, 2008/09) or 800 (2006/07, 2007/08). Independent of licence type, the number of diarists in the initial PDS of 2000/01 was chosen to restrict sampling error on the estimated probability that a licensee did fish for lobster ($p_i^{(pds)}$), to be < 10% (Baharthah 2007). The relative differences between $\hat{L}_{t,v}^{(m)}$ and $\hat{L}_{t,v}^{(pds)}$ (35.2 - 85.4,
Table 3.3) being much more stable than those between \( \hat{N}_{t,v}^{(m)} \) and \( \hat{N}_{t,v}^{(pds)} \) \((-7.6 – 175.7, Table 3.3) when using only 400 diarists is likely a result of this control. In the two seasons that 800 diarists were enrolled, the range on the relative difference between \( \hat{N}_{t,v}^{(m)} \) and \( \hat{N}_{t,v}^{(pds)} \) (70.2 – 105.6, Table 3.3) was much shorter although it must be noted that this range is based on only two seasons of data. Future numbers of diarists should be chosen to also control the sampling error on both \( \hat{N}_{t,v}^{(pds)} \) and \( \hat{N}_{t,v}^{(pds)} \).

Although they reflected the abundance of lobster, the scale of \( \hat{N}_{t,v}^{(m)} \) and \( \hat{N}_{t,v}^{(pds)} \) differed markedly and has a slightly larger impact on the difference between \( \hat{N}_{t,v}^{(m)} \) and \( \hat{N}_{t,v}^{(pds)} \) than that explained by \( \hat{T}_{t,v}^{(m)} \) and \( \hat{T}_{t,v}^{(pds)} \). On further investigation it was seen that for a given level of reported number of days fished, the estimates of \( \hat{F}_{t,v}^{(m)} \) and \( \hat{F}_{t,v}^{(pds)} \) were actually similar and their overall difference was due to an avidity bias where a significantly greater proportion of people in the MS reported higher numbers of days fished than in the PDS, for both potting and diving.

In a study comparing a MS and PDS of anglers who fished Lake Ontario in 1991 and 1992, Connelly & Brown (1995) concluded that fishers were more likely to accurately recall their total catches for the previous season but significantly overestimated the number of days fished. This study, where respondents of the MS could be paired to a PDS of the same person, also demonstrated that whilst fishers were able to accurately recall a ‘low’ number of days fished, fishers were more likely to overestimate the number of days fished in the MS as their actual number of days fished (taken from the PDS) increased. This suggests that the western rock lobster MS reporting a higher proportion of avid fishers than the PDS could partly be due to some people overestimating the days they fished.

Plots of \( \hat{L}_{t,v}^{(s)} \) and \( \hat{F}_{t,v}^{(s)} \) for each fishing and survey method showed that there is a consistent relationship for each of these between survey methods and supports the concept of using a correction factor to standardize \( \hat{N}_{t,v}^{(m)} \) to \( \hat{N}_{t,v}^{(pds)} \) for seasons when only the MS has been performed. Under the assumption that \( \hat{N}_{t,v}^{(pds)} \) is more accurate, this standardization will correct \( \hat{N}_{t,v}^{(m)} \) for the nonresponse bias of people who fished.
being more likely to return the survey form, the avidity bias of more avid fishers being more likely to return a survey form than those less avid, and a likely recall bias where for example, people are less likely to accurately recall the number of days fished.

Independent of fishing method, separate correction factors standardizing \( \hat{T}_{t,v}^{(m)} \) to \( \hat{T}_{t,v}^{(pds)} \) (0.66, s.e. = 0.01) and \( \hat{F}_{t,v}^{(m)} \) to \( \hat{F}_{t,v}^{(pds)} \) (0.59, s.e. = 0.05) were firstly identified. Multiplying these, the correction factor for standardizing \( \hat{T}_{t,v}^{(m)} \) to \( \hat{T}_{t,v}^{(pds)} \) (0.39, s.e. = 0.03) was calculated. A comparison between a telephone recall survey (a six month recall period) and PDS of recreational fishers in Tasmania for three different periods (Lyle 1999) reported that the PDS estimates were 0.2 – 0.8 times those of the corresponding recall surveys. The conversion ratio of 0.39 estimated here for the western rock lobster MS falls centrally within this range.

With possible exception to seasons where the average retained catch per fisher (Figure 3.21) were flagged as outliers (2000/01, 2004/05 and 2008/09), applying the estimated correction factor of 0.39 to \( \hat{T}_{t,v}^{(m)} \) resulted in catch estimates that were similar to those of the PDS. Which survey method is responsible for producing the outliers in the average catch per fisher estimates is unclear, although it is noted that these outliers correspond to seasons where the PDS enrolled only 400 diarists. Although they were similar, there was evidence that the relative difference between the PDS and corrected MS estimates were correlated with seasonal changes in the distribution of days fished as estimated by the PDS.

This research has demonstrated that the level of bias in total catch estimated from a MS of recreational anglers is dependent on differences in probabilities of returning the survey form for people with different fishing frequencies. Given these probabilities are likely to vary from fishery to fishery it should not be expected that a single correction factor will be generally applicable to all. In fact, although the identified correction factor may be appropriate for the recreational western rock lobster fishery now, it may not be in the future. For example, in reaction to the implementation of some unpopular fishing rule by fisheries managers to restrict total catches in the future, avid fishers may become less likely to return the survey form.
The relative difference of the corrected MS estimates to those of the PDS was seen to be correlated with seasonal changes in the distribution of days fished. Hence, even if response probabilities were to remain constant over time, the possibility of effort distributions varying between seasons supports the frequent intermittent running of a PDS (in parallel to a MS) to assess the continued adequacy of a correction factor. The seasons in which both surveys should both be run will be determined by factors that are suspected to affect the probability of fishers returning the survey (e.g. run both surveys for seasons in which a new management change to restrict recreational catch is introduced) or their distribution of days fished (e.g. people expected to fish more/less days due to a prediction of unusually high/low catches for the upcoming season).

Motivated by the observed bias in the recreational western rock lobster fishery that people who fished for rock lobster are more likely to return the survey form, Chapter 4 identifies an estimator for better estimating the proportion of a surveyed population that has some characteristic, in the presence of such a bias. This will aid in generalizing the identification of a correction factor that can be used by other fisheries to reduce the bias in catch estimates from a MS, and will help reduce the need to run expensive surveys such as the PDS.
Chapter 4

Identifying Estimators that Reduce the Impact of Nonresponse Bias on the Estimate of a Dichotomous Variable

Assuming that the phone-diary survey (PDS) is less biased, Chapter 3 adjusted the mail survey (MS) derived total catch estimates of the recreational western rock lobster fishery for nonresponse, recall and avidity bias by applying an identified correction factor that standardized these estimates to that expected if using a PDS. To reduce the need of running more accurate but usually more expensive surveys such as the PDS, this Chapter identifies estimators that can be used to identify that part of the correction factor that corrects for nonresponse and avidity (but not recall) bias.

In comparing responses of the MS to the PDS in Chapter 3, it was seen that the probability of a person returning the survey form if they fished ($\pi_f$) is greater than those that did not ($\pi_{\bar{f}}$) i.e. $\pi_f > \pi_{\bar{f}}$. This leads to the MS total catch estimates having a higher representation of avid fishers than the corresponding PDS and hence, much higher catch estimates. Total catch estimates from the MS have traditionally been derived using the ratio estimator ($\hat{\gamma}_r$), which estimates the proportion of people who fished in a particular season as the proportion of respondents to the survey who reported having fished.

The $\hat{\gamma}_r$ is used to estimate the proportion of a population ($\gamma$) that has some characteristic ($x$) under the assumption that the probability of responding for someone with the characteristic ($\pi_x$) and someone without ($\pi_{\bar{x}}$) is equal ($\pi_x = \pi_{\bar{x}}$). Motivated by the observation in Chapter 3 and support from other fields that highly ‘involved’ or ‘interested’ people are more likely to return a survey than those who are not (e.g. $\pi_f > \pi_{\bar{f}}$) this Chapter constructs two competing estimators to $\hat{\gamma}_r$: the expectation ($\hat{\gamma}_e$) and multinomial ($\hat{\gamma}_m$) estimator, that differ in their underlying values of $\pi_x$ and $\pi_{\bar{x}}$. Comparing these estimators in terms of root-mean-square-error
(RMSE), a commonly used measure to rank competing estimators, comparisons show that on average, \( \hat{\gamma}_m \) outperforms \( \hat{\gamma}_o \) and \( \hat{\gamma}_c \) when \( \gamma \leq 0.65 \). Over this range it is seen that \( \hat{\gamma}_o \) produces an average percentage bias of between 25% and 131% and \( \hat{\gamma}_m \) reduces these by between 53% and 100%. Over the same range, using \( \hat{\gamma}_c \) leads to reductions of between 48% and 61%. Although the percentage reduction in bias of \( \hat{\gamma}_c \) is lower than that achieved by \( \hat{\gamma}_m \) when \( \gamma \leq 0.65 \), it does have the advantage of not running the risk of increasing the bias. For \( \gamma > 0.65 \), \( \hat{\gamma}_m \) increases the average relative bias in \( \hat{\gamma}_o \) by as much as 333% whilst \( \hat{\gamma}_c \) continues to reduce, although the achieved reductions in actual bias in this range are small.

Calculating \( \hat{\gamma}_m \) and \( \hat{\gamma}_o \) and their variance for an observed sample is simple. Calculating \( \hat{\gamma}_c \) however requires the calculation of two additional parameters that lead to formulaic approximations of its variance being much more complex than for \( \hat{\gamma}_o \) and \( \hat{\gamma}_m \). Bootstrapping is successfully used to overcome this. The additional computing requirements to calculate the estimate of \( \hat{\gamma}_c \) and its variance are not considered onerous.

This Chapter gives some background information and definitions useful to what follows and then constructs two competing estimators, \( \hat{\gamma}_c \) and \( \hat{\gamma}_m \), to \( \hat{\gamma}_o \). The variance and confidence interval procedures are then identified for each of these estimators. Finally, the ability of all three estimators to accurately estimate \( \gamma \) for various levels of \( \pi_x \) and \( \pi_{\bar{x}} \) are compared.

### 4.1 Background

Due to its ease of use and low cost, the MS method has commonly been used to collect information for estimating the proportion \( (\gamma) \) of a population that has some characteristic \( x \). The MS method generally involves randomly choosing \( w \) members (assumed adjusted for ‘return to sender’ throughout) from an identified population of size \( W \) and sending them a survey form. The returned forms are then summarized as the number reporting having \( x \) \( (w_x) \) and not \( (w_{\bar{x}}) \). The \( \hat{\gamma}_o \) assumes that the probability of responding for a member with \( x \) \( (\pi_x) \) and someone without \( (\pi_{\bar{x}}) \) is equal \( (\pi_x = \pi_{\bar{x}}) \)
and estimates $\gamma$ as the proportion of respondents who reported having this characteristic i.e. $\hat{\gamma}_o = \frac{w_x}{w_x + w_o}$. If $\pi_x > \pi_{\bar{x}}$ then applying $\hat{\gamma}_o$ leads to a positively biased estimate of $\gamma$.

Many methods have been used to adjust for nonresponse bias (as discussed in Chapter 2). These methods generally assume that responses for nonrespondents can be predicted by those of respondents modelled by auxiliary data (AD) known for all survey recipients. These methods however can give false confidence that resulting estimates have been properly adjusted for nonresponse, particular if important AD have been omitted, or are not available for both respondents and nonrespondents, and when the AD is not predictive for both response propensity and the response variable being adjusted. In fact, it is even possible to further bias the results compared to no adjustment at all. This Chapter deals with nonresponse bias by identifying estimators that better estimate the mean of a dichotomous variable such as $\gamma$ in the presence of nonresponse bias when $\pi_x > \pi_{\bar{x}}$ and do not rely on the use of AD. The definitions for symbols used throughout this Chapter are summarized in Table 4.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>The probability that a member in the studied population has characteristic $x$.</td>
</tr>
<tr>
<td>$\hat{\gamma}_o$</td>
<td>The $y$ estimator for estimating $\gamma$.</td>
</tr>
<tr>
<td>$W$</td>
<td>The size of the population for which $\gamma$ is to be estimated.</td>
</tr>
<tr>
<td>$w$</td>
<td>The number of members in the population sent a survey form.</td>
</tr>
<tr>
<td>$\pi_x$</td>
<td>The probability that a member with characteristic $x$ will return the survey form.</td>
</tr>
<tr>
<td>$\pi_{\bar{x}}$</td>
<td>The probability that a member without characteristic $x$ will return the survey form.</td>
</tr>
<tr>
<td>$w_x$</td>
<td>The number of returned surveys that reported having characteristic $x$.</td>
</tr>
<tr>
<td>$w_{\bar{x}}$</td>
<td>The number of survey forms returned that reported not having characteristic $x$.</td>
</tr>
<tr>
<td>$x$</td>
<td>A particular characteristic for which the presence in the population is being studied e.g. did you fish?</td>
</tr>
</tbody>
</table>
4.2 The Ratio Estimator $\hat{\gamma}_o$

The ratio estimator $\hat{\gamma}_o = \frac{w_x}{w_x + w_{\bar{x}}}$ is a maximum likelihood estimator (MLE) of $\gamma$ where the probability of observing $w_x$ and $w_{\bar{x}}$ is assumed to be a realisation from the binomial distribution $B(w_x; w_x + w_{\bar{x}}, \gamma)$. In determining $\hat{\gamma}_o$ the nonrespondents are ignored and the sample size assumed to be $w_x + w_{\bar{x}}$ and not $w$. When $w = w_x + w_{\bar{x}}$ it can be shown, using the properties of the binomial distribution, that $\sigma^2_{\hat{\gamma}_o} = \frac{\gamma(1-\gamma)}{w_x + w_{\bar{x}}}$ and is estimated by $\hat{\sigma}^2_{\gamma_o} = \frac{\hat{\gamma}(1-\hat{\gamma})}{w_x + w_{\bar{x}}}$. Under the assumption that $\pi = \pi_{\bar{x}}$ then $\sigma^2_{\gamma_o} > \frac{\gamma(1-\gamma)}{w_x + w_{\bar{x}}}$ and a more accurate $\hat{\sigma}^2_{\gamma_o}$ should be identified.

Using Taylor series expansions and assuming $E(S) \neq 0$, Kendall & Stuart (1963) approximate the mean and variance of the ratio of two random variables $R$ and $S$ as:

$$E\left(\frac{R}{S}\right) \approx \frac{E(R)}{E(S)} - \frac{Cov(R,S)}{E^2(S)} + \frac{Var(S)E(R)}{E^3(S)} \quad (2^{nd} \text{order Taylor series approx})$$

and

$$Var\left(\frac{R}{S}\right) \approx \frac{E^2(R)}{E^2(S)} \left( \frac{Var(R)}{E^2(R)} + \frac{Var(S)}{E^2(S)} - \frac{2Cov(R,S)}{E(R)E(S)} \right) \quad (1^{st} \text{order Taylor series approx})$$

$E\left(\frac{R}{S}\right)$ and $Var\left(\frac{R}{S}\right)$ have been approximated by a second and first order Taylor series expansion of $R/S$, respectively. The order of expansion used to approximate each of these quantities has been chosen due to their resulting simplicity. Higher order expansions lead to more complex formulas without necessarily much improvement in accuracy. Simulations will be used to assess the adequacy of these approximations (to be described further on).

Substituting $R = W_x$ and $S = W_{\bar{x}} + W_x$ the expected value and variance of $\hat{\gamma}_o$ is approximated as:

$$E(\hat{\gamma}_o) \approx \frac{E(W_x)}{E(W_x + W_{\bar{x}})} - \frac{Cov(W_x,W_{\bar{x}} + W_x)}{E^2(W_x + W_{\bar{x}})} + \frac{Var(W_{\bar{x}} + W_x)E(W_x)}{E^3(W_x + W_{\bar{x}})}$$

and

$$\sigma^2_{\hat{\gamma}_o} \approx \frac{E^2(W_x)}{E^2(W_x + W_{\bar{x}})} \left( \frac{Var(W_x)}{E^2(W_x)} + \frac{Var(W_{\bar{x}} + W_x)}{E^2(W_x + W_{\bar{x}})} - \frac{2Cov(W_x,W_{\bar{x}} + W_x)}{E(W_x)E(W_{\bar{x}} + W_x)} \right).$$
Using estimates of the required inputs for these formula explaining $\mu_{\gamma_o}$ and $\sigma^2_{\gamma_o}$ leads to estimates $\hat{\mu}_{\gamma_o}$ and $\hat{\sigma}^2_{\gamma_o}$ respectively. Assuming $(W_x, W_{\bar{x}}) \sim M(X; w, \hat{\theta})$, where $M(\cdot)$ is the multinomial distribution with $X = (w_x, w_{\bar{x}}, w - w_x - w_{\bar{x}})$ and $\hat{\theta} = (\frac{w_x}{w}, \frac{w_{\bar{x}}}{w}, \frac{w - w_x - w_{\bar{x}}}{w})$, the estimated inputs required for calculating $\hat{\mu}_{\gamma_o}$ and $\hat{\sigma}^2_{\gamma_o}$ are:

\[
\begin{align*}
E(W_x) &= w_x, \\
E(W_{\bar{x}} + W_x) &= w_{\bar{x}} + w_x, \\
\text{Var}(W_x) &= w_x(1 - \frac{w_x}{w}), \\
\text{Var}(W_{\bar{x}} + W_x) &= w_{\bar{x}}(1 - \frac{w_x}{w}) + w_x(1 - \frac{w_{\bar{x}}}{w}) - \frac{2w_xw_{\bar{x}}}{w}, \\
\text{Cov}(W_{\bar{x}}, W_x) &= -\frac{w_xw_{\bar{x}}}{w} \quad \text{and} \\
\text{Cov}(W_x, W_{\bar{x}} + W_x) &= \text{Cov}(W_{\bar{x}}, W_x) + \text{Var}(W_x).
\end{align*}
\]

The bias in $\hat{\gamma}_o$ for estimating $\gamma (b_j(\hat{\gamma}_o) = E(\hat{\gamma}_o) - \gamma)$ is dependent on $\pi_x$ and $\pi_{\bar{x}}$. Having ignored nonrespondents in the identification of $\hat{\gamma}_o$ it follows that $b_j(\hat{\gamma}_o) = 0$ if $\pi_x = \pi_{\bar{x}}$; else, $b_j(\hat{\gamma}_o) \neq 0$.

### 4.3 Alternative estimators to $\hat{\gamma}_o$

Given $\hat{\gamma}_o$ is a biased estimator of $\gamma$ when $\pi_x \neq \pi_{\bar{x}}$, attempts are made to identify estimators that although they still may be biased, are less so than this traditionally used estimator. To aid in identifying such alternatives, $\hat{\gamma}_o$ is firstly rearranged and solved in terms of $\gamma$.

Substituting $E(w_x) = w\gamma \pi_x$ and $E(w_{\bar{x}}) = w(1 - \gamma)\pi_{\bar{x}}$ into $\hat{\gamma}_o = \frac{w_x}{w_{\bar{x}} + w_x}$, for $w_x$ and $w_{\bar{x}}$ respectively, leads to:

\[
\hat{\gamma}_o = \frac{w\gamma \pi_x}{w(1 - \gamma)\pi_{\bar{x}} + w\gamma \pi_x}.
\]

Rearranging,

\[
[w(1 - \gamma)\pi_{\bar{x}} + w\gamma \pi_x] \hat{\gamma}_o = w\gamma \pi_x
\]
and dividing each side by the common factor $w$,

$$[\hat{p}_x - \gamma \hat{p}_x + \gamma \hat{p}_x] \gamma_0 = \gamma \hat{p}_x .$$

Expanding and grouping like terms,

$$[\hat{p}_x - (\hat{p}_x - \hat{p}_x) \gamma_0] \gamma = \hat{p}_x \gamma_0$$

and so

$$\gamma = \frac{\hat{p}_x \gamma_0}{\hat{p}_x - (\hat{p}_x - \hat{p}_x) \gamma_0} .$$

Making use of the preceding description, the multinomial ($\hat{\gamma}_m$) and expectation ($\hat{\gamma}_e$) estimators are presented as alternatives to $\hat{\gamma}_0$ for estimating $\gamma$ and differ in their estimate of $\hat{p}_x$ and $\hat{p}_x$ ($\hat{\gamma}_o$ sets $\hat{p}_x = \hat{p}_x$). Approximate variance functions for these estimators are also presented.

**The “multinomial-estimator” $\hat{\gamma}_m$**

The multinomial estimator ($\hat{\gamma}_m$) is constructed assuming $\hat{p}_x = \frac{w_p}{w \gamma_m}$ and $\hat{p}_x = \frac{w_p}{w(1-\gamma_m)}$. Given the assumption that $\hat{p}_x > \hat{p}_x$, the Manski’s Bounds (see Chapter 2) on $\gamma$ are given by $\gamma \in \left[ \frac{w_p}{w}, \hat{\gamma}_0 \right]$.

The root-mean-square-error (RMSE) for the estimator $\hat{\theta}$ of $\theta$ is defined as:

$$\text{RMSE}(\hat{\theta} | \theta) = \sqrt{\frac{1}{j} \sum_{i=1}^{j} (\hat{\theta}_i - \theta)^2} ,$$

where $j$ is the total number of estimates $\hat{\theta}$ for $\theta$.

The RMSE is commonly used to choose between competing estimators of $\theta$ with that estimator which minimizes this criterion being preferred (Walther & Moore 2005;
Kreuter & Olsen (2011). Assuming that $\gamma$ is uniformly distributed over $\gamma \in \left[ \frac{w_x}{\gamma_m}, \frac{w_x}{\gamma_o} \right]$, $\hat{\gamma}_m$ is determined to minimize the expected RMSE of $\gamma = \frac{\pi_x \hat{\gamma}_o}{\pi_x - (\pi_x - \pi_y) \hat{\gamma}_o}$, where

$$\pi_x = \frac{w_x}{w \gamma_m} \text{ and } \pi_y = \frac{w_y}{w (1 - \gamma_m)}$$ i.e.

$$\frac{d}{d\hat{\gamma}_m} \int_{\frac{w_x}{w \gamma_m}}^{\frac{w_x}{w \gamma_o}} \frac{1}{\hat{\gamma}_o - \frac{w_x}{w \gamma_m} \left( \frac{w_y}{w \gamma_m} - \frac{w_y}{w (1 - \gamma_m)} \right) \hat{\gamma}_o - \gamma}^2 d\gamma = 0,$$

or equivalently,

$$\frac{d}{d\hat{\gamma}_m} \int_{\frac{w_x}{w \gamma_m}}^{\frac{w_x}{w \gamma_o}} \frac{1}{\hat{\gamma}_m - \frac{w_x}{w \gamma_o} (\hat{\gamma}_m - \gamma)^2} d\gamma = 0.$$

It is easily verified that $\hat{\gamma}_m = \frac{1}{2} \left( \frac{w_x}{w} + \hat{\gamma}_o \right)$ solves the above equation.

Using the known relationship

$$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2ab Cov(X, Y):$$

it follows that:

$$Var(\hat{\gamma}_m) = Var\left( \frac{1}{2} \left( \frac{w_x}{w} + \hat{\gamma}_o \right) \right) = \frac{1}{4} \left( \frac{Var(W_x)}{w^2} + Var(\hat{\gamma}_o) + 2 Cov(\frac{W_x}{w}, \hat{\gamma}_o) \right),$$

where $Var(W_x)$ and $Var(\hat{\gamma}_o)$ are estimated as previously.

To simplify estimation of $Cov(\frac{W_x}{w}, \hat{\gamma}_o)$, $\hat{\gamma}_o$ is approximated using a second order Taylor series expansion at the point $W_x = w_x$ and $W_y = w_y$ i.e.

$$\hat{\gamma}_m(W_x, W_y) \approx \hat{\gamma}_m(w_x, w_y) + (W_x - w_x) \frac{\partial}{\partial W_x} \hat{\gamma}_m(w_x, w_y) + (W_y - w_y) \frac{\partial}{\partial W_y} \hat{\gamma}_m(w_x, w_y)$$

$$\approx \frac{w_x}{w_y + w_x} + (W_x - w_x) \frac{w_y}{(w_y + w_x)^2} - (W_y - w_y) \frac{w_y}{(w_y + w_x)^2}.$$
Therefore:

\[
\text{Cov}(\frac{W_x}{w}, \hat{\gamma}_o) \approx \text{Cov}(\frac{W_x}{w}, \hat{\gamma}_m(W_x, W_x))
\]

\[
\approx \text{Cov}\left(\frac{W_x}{w}, \frac{w_x}{w_y+w_x} + (W_x - w_x)\frac{w_x}{(w_y+w_x)^2} - (W_x - w_x)\frac{w_x}{(w_y+w_x)^2}\right)
\]

\[
\approx \frac{1}{w_y+w_x}\text{Cov}(W_x, w_yW_x - w_xW_x),
\]

where \(\text{Var}(W_x)\) and \(\text{Cov}(W_x, W_x)\) are estimated as previously.

**The “expectation-estimator” \(\hat{\gamma}_e\)**

The expectation estimator \((\hat{\gamma}_e)\) is constructed by assuming

\[
\pi_i = E[\Pi_i|\Pi_x \geq \Pi_x], i = \bar{x}, x \quad \text{i.e.}
\]

\[
\hat{\pi}_e = \frac{E[\Pi_x|\Pi_x \geq \Pi_x] \hat{\gamma}_o}{E[\Pi_x|\Pi_x \geq \Pi_x] - (E[\Pi_x|\Pi_x \geq \Pi_x] - E[\Pi_x|\Pi_x \geq \Pi_x])\hat{\gamma}_o}.
\]

Assuming that the response parameters are uniformly distributed over their respective Manski’s Bound (i.e. \(\Pi_i \sim U(g_i, 1)\) where \(g_i = \frac{w_i}{w_i - w_i^*}\) for \(i = \bar{x}, x, \) and \(\bar{x} = x)\), the calculation of values for \(E[\Pi_x|\Pi_x > \Pi_x]\) and \(E[\Pi_x|\Pi_x > \Pi_x]\) depend on the comparative magnitude of the lower bounds on \(\Pi_x\) and \(\Pi_x\) (see Appendix C for all workings):

**Case i)** Lower bound on \(\Pi_x\) is at least the magnitude for that on \(\Pi_x\) i.e. \(q_{\bar{x}} > q_x\):

\[
E[\Pi_x|\Pi_x > \Pi_x] = \frac{1}{|A|} \left(\frac{1}{3} - \frac{q_x}{2} + \frac{q_x^2}{6}\right)
\]

and

\[
E[\Pi_{\bar{x}}|\Pi_x > \Pi_{\bar{x}}] = \frac{1}{|A|} \left(\frac{1}{6} - \frac{q_{\bar{x}}^2}{2} + \frac{q_{\bar{x}}^3}{3}\right),
\]

where \(|A| = \frac{(1-q_x)^2}{3} \) and
Case ii) Lower bound on $\Pi_x$ is smaller in magnitude than that on $\Pi_x$ i.e. $g_{x^c} < g_x$.

$$
E[\Pi_x | \Pi_x > \Pi_x] = \frac{1}{|B|} \left( \frac{1}{3} - \frac{g_x}{2} - \frac{g_x^2}{2} \left( \frac{g_x}{3} - \frac{g_x}{2} \right) \right) \text{ and }
$$

$$
E[\Pi_x | \Pi_x > \Pi_x] = \frac{1}{|B|} \left( -\frac{g_x^2 - g_x^3 + g_x \sigma_y^2}{2} + \frac{1}{6} + \frac{g_x^3}{3} \right),
$$

where $|B| = (1 - g_x)(g_x - g_{x^c}) + \frac{(1-g_x)^2}{2}$.

The expected values $E[\Pi_x | \Pi_x > \Pi_x]$ and $E[\Pi_x | \Pi_x > \Pi_x]$ vary depending on the observed values $w_x$ and $w_{x^c}$ which leads to the first order Taylor series expansion estimate of $\sigma_{\gamma_x}^2$ being vastly more complicated than that for $\sigma_{\gamma_m}^2$ and $\sigma_{\gamma_m}^2$. It was therefore decided to use bootstrapping (Efron & Tibshirani 1986; MacKinnon 2006) to approximate the variance. Efron & Tibshirani (1986) suggest that using at most 200 simulations is sufficient for achieving a good approximation. Hesterberg (2011) however, suggests that the required number may be much higher than this. For the purpose of illustrating the effectiveness of $\widehat{\gamma}_x$ over various ranges of $\gamma$, $\pi_x$ and $\pi_{x^c}$, using 200 simulations was considered sufficient.

### 4.4 Constructing Confidence Intervals for Estimators of $\gamma$

The $100(1 - \alpha)\%$ confidence interval for the estimate $\widehat{\gamma}_y$ of $\gamma$ is defined by $(L_\alpha(\widehat{\gamma}_y), U_\alpha(\widehat{\gamma}_y))$ such that $\Pr(L_\alpha(\widehat{\gamma}_y) \leq E(\widehat{\gamma}_y) \leq U_\alpha(\widehat{\gamma}_y)) = 1 - \alpha$ (Walpole & Myers 1990). There are many methods in the literature to estimate confidence intervals for a binomial proportion (e.g. Casella 1986; Böhning 1994; Blaker 2000). A method often used as the basis of many of these is the Clopper-Pearson (CP) confidence interval. The CP confidence interval is popularly known as an “exact” confidence interval since it is based on the quantiles of the binomial distribution and not some approximation such as the Wald interval that uses a normal distribution approximation (Tobi et al. 2005).
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Assuming that \( Z \sim B(z; h, \gamma) \) the CP interval is constructed such that
\[
\begin{align*}
  z_l &= \max\{ z | P(Z \geq z) \geq 1 - \frac{\alpha}{2} \} \quad \text{and} \quad z_u = \min\{ z | P(Z \geq z) \leq \frac{\alpha}{2} \}.
\end{align*}
\]
The observed confidence level of \( \gamma \) is then given by \((L_\alpha(\widehat{\gamma}), U_\alpha(\widehat{\gamma})) = \left( \frac{z_u}{h}, \frac{z_l}{h} \right)\). Since the binomial distribution is discrete however, the choice of \( z_l \) and \( z_u \) leads to the confidence of this interval being \( \geq 1 - \alpha \) and hence, the CP interval being conservative i.e. too wide (Brown et al. 2001; Liu & Kott 2007).

The bounds of the CP 100(1−\( \alpha \))% can be determined exactly (Leemis & Trivedi 1996) and are given by:
\[
(1 + \frac{h-z+1}{zF(\frac{1}{2},2z;2(h-z+1))})^{-1} < \gamma < (1 + \frac{h-z}{(z+1)F(1-\frac{1}{2},2(z+1),2(h-z))})^{-1},
\]
where \( F(c, v_1, v_2) \) is the c quantile of the Snedecor F distribution with \( v_1 \) and \( v_2 \) degrees of freedom (Dudewicz & Mishra 1988) and \( z \) is the number of successes reported from the data.

The CP interval was derived for \( \gamma_y \) assuming that \( z \) is a realisation from a binomial distribution under no nonresponse. To use this method to estimate the 100(1−\( \alpha \))% confidence interval for \( \gamma_y \) under nonresponse, \( h \) and observed \( z \) need to be adjusted so that the variance of the binomial distribution is equal to the variance of \( \gamma_y \). Sowmya et al. (2004) show that setting \( h = h^* \), where \( h^* = \frac{\widehat{\gamma}_y(1-\widehat{\gamma}_y)}{\sigma_{\widehat{\gamma}_y}^2} \) and \( z = h^* \widehat{\gamma} \), achieves this.

### 4.5 Criteria for Identifying ‘Best’ of Competing Estimators

When comparing two estimators \( \hat{\Theta}_1 \) and \( \hat{\Theta}_2 \) it is intuitively reasonable that the optimal estimator is that which has the minimum bias \( b(\hat{\Theta}|\Theta) \) (accuracy: the estimator is “closer” to the parameter it is estimating) and minimum variance \( \text{Var}(\hat{\Theta}) \) (precision: the estimate does not change drastically from sample to sample). Since two competing estimators may be more optimal in one of these characteristics and
not the other, the root-mean-square-error (RMSE) is commonly used to determine
the better of two competing estimators:

\[
\text{RMSE}(\hat{\theta}|\theta) = \sqrt{\frac{1}{j} \sum_{i=1}^{j} (\hat{\theta}_i - \theta)^2},
\]

where \( j \) is the total number of estimates for \( \theta \).

Casella & Berger (1990) showed that \( \text{RMSE}(\hat{\theta}|\theta) = \sqrt{b(\hat{\theta}|\theta)^2 + \text{Var}(\hat{\theta})} \) and hence, the RMSE is a function of the two important measures of an estimator previously
described.

The RMSE will be the measure used throughout to determine which of the
competing estimators is “best” under different scenarios of \( \gamma, \pi_v \) and \( \pi_w \). To
determine when estimator \( \hat{\theta}_2 \) is a ‘better’ estimator of \( \theta \) than \( \hat{\theta}_1 \), lemma 1 is
presented.

Lemma 1: For two estimators \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \), the conditions that ensure
\( \text{RMSE}(\hat{\theta}_2) \leq \text{RMSE}(\hat{\theta}_1) \) when estimating \( \theta \) are:

i) \( \hat{\theta}_1 > \theta, \hat{\theta}_2 > \theta \) and \( \hat{\theta}_1 > \hat{\theta}_2 \); or

ii) \( \hat{\theta}_1 > \theta, \hat{\theta}_2 < \theta \) and \( \hat{\theta}_1 \leq \frac{\hat{\theta}_1 + \hat{\theta}_2}{2} \); or

iii) \( \hat{\theta}_1 < \theta, \hat{\theta}_2 \geq \theta \) and \( \theta \geq \frac{\hat{\theta}_1 + \hat{\theta}_2}{2} \); or

iv) \( \hat{\theta}_1 < \theta, \hat{\theta}_2 < \theta \) and \( \hat{\theta}_1 < \hat{\theta}_2 \).

Proof:
The condition

\[
\text{RMSE}(\hat{\theta}_2) \leq \text{RMSE}(\hat{\theta}_1)
\]

is equivalent to

\[
(\hat{\theta}_2 - \theta)^2 \leq (\hat{\theta}_1 - \theta)^2.
\]
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There are 4 cases of $\hat{\theta}_1 - \theta$ and $\hat{\theta}_2 - \theta$ to be considered in determining the relationship between $\hat{\theta}_1$ and $\hat{\theta}_2$ that ensures the above condition holds:

i) $\hat{\theta}_1 - \theta > 0$ and $\hat{\theta}_2 - \theta > 0$;
ii) $\hat{\theta}_1 - \theta > 0$ and $\hat{\theta}_2 - \theta < 0$;
iii) $\hat{\theta}_1 - \theta < 0$ and $\hat{\theta}_2 - \theta \geq 0$; and
iv) $\hat{\theta}_1 - \theta < 0$ and $\hat{\theta}_2 - \theta < 0$.

Case i)
Assuming $\hat{\theta}_1 - \theta \geq 0$ and $\hat{\theta}_2 - \theta \geq 0$ means that the values of $\theta$ for which $(\hat{\theta}_2 - \theta)^2 \leq (\hat{\theta}_1 - \theta)^2$ hold are the same as for the condition $\hat{\theta}_1 - \theta < \hat{\theta}_2 - \theta$. Solving this condition leads to $\hat{\theta}_1 > \hat{\theta}_2$. Therefore, if $\hat{\theta}_1 > \theta, \hat{\theta}_2 > \theta$ (equivalent conditions for $\hat{\theta}_1 - \theta \geq 0$ and $\hat{\theta}_2 - \theta \geq 0$, respectively) and $\hat{\theta}_1 \geq \hat{\theta}_2$ then $\text{RMSE}(\hat{\theta}_2) \leq \text{RMSE}(\hat{\theta}_1)$.

Case ii)
Assuming $\hat{\theta}_1 - \theta \geq 0$ and $\hat{\theta}_2 - \theta < 0$ means that the values of $\theta$ for which $(\hat{\theta}_2 - \theta)^2 \leq (\hat{\theta}_1 - \theta)^2$ hold are the same as for the condition $\hat{\theta}_1 - \theta \leq \hat{\theta}_2 - \theta$ ($\hat{\theta}_2 - \theta$ has been replaced by $\theta - \hat{\theta}$, to keep differences positive and removing need to square). Solving this condition leads to $\theta \leq \frac{\hat{\theta}_1 + \hat{\theta}_2}{2}$. Therefore, if $\hat{\theta}_1 > \theta, \hat{\theta}_2 < \theta$ and $\theta \leq \frac{\hat{\theta}_1 + \hat{\theta}_2}{2}$ then $\text{RMSE}(\hat{\theta}_2) \leq \text{RMSE}(\hat{\theta}_1)$.

Case iii)
Assuming $\hat{\theta}_1 - \theta < 0$ and $\hat{\theta}_2 - \theta > 0$ means that the values of $\theta$ for which $(\hat{\theta}_2 - \theta)^2 \leq (\hat{\theta}_1 - \theta)^2$ hold are the same as for the condition $\hat{\theta}_2 - \theta \leq \theta - \hat{\theta}_1$. Solving this condition leads to $\theta \geq \frac{\hat{\theta}_1 + \hat{\theta}_2}{2}$. Therefore, if $\hat{\theta}_1 < \theta, \hat{\theta}_2 \geq \theta$ and $\theta \geq \frac{\hat{\theta}_1 + \hat{\theta}_2}{2}$ then $\text{RMSE}(\hat{\theta}_2) \leq \text{RMSE}(\hat{\theta}_1)$.

Case iv)
Assuming $\hat{\theta}_1 - \theta < 0$ and $\hat{\theta}_2 - \theta < 0$ means that the values of $\theta$ for which $(\hat{\theta}_2 - \theta)^2 \leq (\hat{\theta}_1 - \theta)^2$ hold are the same as for the condition $\theta - \hat{\theta}_2 \leq \theta - \hat{\theta}_1$. Solving
this condition leads to $\hat{\theta}_1 < \hat{\theta}_2$. Therefore, if $\hat{\theta}_1 < \theta$, $\hat{\theta}_2 < \theta$ and $\hat{\theta}_1 < \hat{\theta}_2$, then $\text{RMSE}(\hat{\theta}_2) \leq \text{RMSE}(\hat{\theta}_1)$.

The RMSE is a function of the estimator’s bias and sampling variance. Thus, two competing estimators can have the same RMSE but exchanged levels for bias and the sampling variance. When tied in terms of RMSE, the estimator with the smallest bias is preferred since the variance can be reduced by increased sampling intensity but the bias can not.

The $100(1 - \alpha)\%$ confidence interval for estimator $\hat{\gamma}_y (L_\alpha(\hat{\gamma}_y), U_\alpha(\hat{\gamma}_y))$, was defined such that $\text{Pr}(L_\alpha(\hat{\gamma}_y) \leq E(\hat{\gamma}_y) \leq U_\alpha(\hat{\gamma}_y)) = 1 - \alpha$. The $\hat{\gamma}_y$ not necessarily being an unbiased estimator of $\gamma$, it is also proposed to assess $\hat{\gamma}_y$ in terms of the probability of $(L_\alpha(\hat{\gamma}_y), U_\alpha(\hat{\gamma}_y))$ to include $\gamma$. This probability will be referred to as the coverage rate of $\gamma$ for the $100(1 - \alpha)\%$ confidence interval of $\hat{\gamma}_y$ i.e. $C_{VT_{\gamma}_y}(\gamma, \alpha) = \text{Pr}(L_\alpha(\hat{\gamma}_y) \leq \gamma \leq U_\alpha(\hat{\gamma}_y))$. For completeness, the probability of the $100(1 - \alpha)\%$ confidence interval of $\hat{\gamma}_y$ to include $E(\hat{\gamma}_y)$ can also be defined as $C_{VT_{\gamma}_y}(E(\hat{\gamma}_y), \alpha)$ which is by construction, equal to $1 - \alpha$.

The range of $\gamma$, $\pi_2$ and $\pi_\infty$ for which $C_{VT_{\gamma}_y}(\gamma, \alpha) > 0$ will be identified for each of the estimators $\hat{\gamma}_o$, $\hat{\gamma}_m$ and $\hat{\gamma}_c$. Whilst comparing RMSE for each of these estimators identifies which performs best under different scenarios, studying when $C_{VT_{\gamma}_y}(\gamma, \alpha) > 0$ identifies when each of these estimators performs ‘well’ in terms of producing observed confidence intervals that contain the true parameter $\gamma$.

In comparing the coverage rate of the confidence interval for each estimator, the width of these intervals will also be compared to ensure that the tendency for one estimator over another to have a higher coverage rate is not at the cost of having a less informative, wider confidence interval.
4.6 Simulations

Important statistics describing the effectiveness of different estimators to estimate \( \gamma \) were simulated. This was accomplished by randomly generating a number of independent random samples of \( w_x \) and \( w_x \) from a population of size \( W \) with each member of the population assumed to be in one of four factions: member with \( x \) that will return a survey \((W \gamma \pi_x)\); member with \( x \) that will not return a survey \((W \gamma (1 - \pi_x))\); member not with \( x \) \((\bar{x})\) that will return a survey \((W (1 - \gamma) \pi_{\bar{x}})\); and member not with \( x \) \((\bar{x})\) that will not return a survey \((W (1 - \gamma) (1 - \pi_{\bar{x}}))\). The random samples of \( w_x \) and \( w_{\bar{x}} \) were then used to calculate the desired statistic(s).

For a given level of \( \gamma \), \( \pi_x \) and \( \pi_{\bar{x}} \), and using inverse binomial cumulation methods (Abramowitz & Stegun 1972), a random sample of \( w_x \) and \( w_{\bar{x}} \) was drawn using Algorithm 1 that constrained \( w_x, w_{\bar{x}} \geq 1 \).

Algorithm 1:

---

**Step 1** Set \( \gamma \), \( \pi_x \) and \( \pi_{\bar{x}} \).

Calculate: \( |X| = W \gamma \) and \( |\bar{X}| = W (1 - \gamma) \).

For fixed \( \gamma \), \( \pi_x \) and \( \pi_{\bar{x}} \) the number members in the surveyed population who have characteristic \( x \) \((X)\) and do not have this characteristic \((\bar{X})\).

**Step 2** Calculate: \( \min |X_w| = \max \{1, w - W (1 - \gamma)\} \) and 
\[ \max |X_w| = \min \{w - 1, W \gamma\} \, . \]

This step calculates the minimum and maximum number of members in \( X \) that can be sent one of the \( w \) surveys \(|X_w|\). It is forced that at least one member in both \( X \) and \( \bar{X} \) receives a survey which means the most that can be sent to members in \( X \) is \( w - 1 \) or \( |X| \). If \( w > |\bar{X}| \) then the most number of surveys that can be sent to \( \bar{X} \) is less than \( w \) which means at least \( w - |\bar{X}| \) surveys need to be sent to members in \( X \).

**Step 3** Calculate: \( \text{low.c.d} = \text{Cbinom}(\min |X_w|; w, \gamma) \) and 
\[ \text{up.c.d} = \text{Cbinom}(\max |X_w|; w, \gamma) \, . \]
This step determines the range of cumulative probability for the binomial distribution that restricts the quantiles to satisfy the valid ranges of $|X_w|$ defined in Step 2.

**Step 4** Randomly draw: \( \text{rprob} = \text{runif}(1, \text{low.cd}, \text{up.cd}) \) and 
\[
|X_w| = \text{qbinom}(\text{rprob}; w, \gamma).
\]
This step randomly chooses a cumulative probability constrained by Step 3 and then its corresponding quantile of the binomial distribution. This gives a random selected number of members in \( X \) that will be sent one of the \( w \) survey forms (\( |X_w| \)).

**Step 5** Calculate: 
\[
\min w_x = \max\{ |X_w| - |X|(1 - \pi_x), 1 \}
\]
\[
\max w_x = \max\{ |X_w|, |X|\pi_x \}
\]
This step calculates the minimum and maximum number of survey forms that will be returned by the \( |X_w| \) members of \( X \) who has been selected to receive a survey (\( w_x \)). If the number of members in \( X \) who will not return a survey (\( |X|(1 - \pi_x) \)) is less than \( |X_w| \) then at least \( |X_w| - |X|(1 - \pi_x) \) members must return the survey. The number of members in \( X \) who will return the survey (\( |X|\pi_x \)) also limits \( w_x \). It is also assumed that at least one member selected to receive the survey will also return the form i.e. \( w_x \geq 1 \).

**Step 6** Calculate:
\[
\text{low.cd} = \text{Chinom}(\min w_x; |X_w|, \pi_x) \quad \text{and}
\]
\[
\text{up.cd} = \text{Chinom}(\max w_x; |X_w|, \pi_x)
\]
\[
\text{rprob} = \text{runif}(1, \text{low.cd}, \text{up.cd})
\]
\[
|X_w| = \text{qbinom}(\text{rprob}; |X_w|, \pi_x)
\]

**Step 7** \( |\overline{X_w}| = w - |X_w| \)
Having determined the number of surveys to be sent to members in \( X \), the number of surveys to be sent to members in \( \overline{X} \) follows.

**Step 8** min \( w_{\overline{x}} = \max\{ |\overline{X_w}| - W(1 - \gamma)(1 - \pi_{\overline{x}}), 1 \} \)
\[
\max w_{\overline{x}} = \max\{ |\overline{X_w}|, W(1 - \gamma)\pi_{\overline{x}} \}
\]
\[
\text{low.cd} = \text{Cbinom}(\min w_{\overline{x}}; |\overline{X_w}|, \pi_{\overline{x}}) \quad \text{and}
\]
\[
\text{up.cd} = \text{Cbinom}(\max w_{\overline{x}}; |\overline{X_w}|, \pi_{\overline{x}})
\]
\[
\text{rprob} = \text{runif}(1, \text{low.cd}, \text{up.cd})
\]
This step randomly selects the number of members in $X$, chosen to receive a survey ($X_w$, **Step 7**), that will return the survey ($w_x$) subject to similar constraints for randomly selecting $w_x$ in **Steps 5 - 6**.

Algorithm 1 is implemented in program-R (R Development Core Team 2008) using various functions that allowed for-loops to be avoided in generating multiple random samples of $w_x$ and $w_{\bar{x}}$ which greatly increases the efficiency of performing the simulations (Appendix D).

For different levels of $\gamma$, $\pi_x$ and $\pi_{\bar{x}}$, the following summary statistics were simulated and graphed to evaluate the performance of estimator $\hat{\gamma}_{u1}$ to a competing estimator $\hat{\gamma}_{w}$: \text{RMSE}(\hat{\gamma}_{u1}) - \text{RMSE}(\hat{\gamma}_{w})$, $\text{CVR}_{\gamma_{u1}}(E(\hat{\gamma}_{u1}), \alpha)$, $\text{CVR}_{\gamma_{w}}(\gamma, \alpha)$, $\sigma_{\hat{\gamma}_{u1}}^2$, $E(\hat{\gamma}_{u1})$ and $E(\hat{\gamma}_{w})$. Each of these statistics were calculated using 5000 randomly drawn samples of $w_x$ and $w_{\bar{x}}$ as described by Algorithm 1. Throughout, population size $W = 40000$ and total surveys sent $n = 4000$ has been used. These values correspond to those that generally describe the recreational western rock lobster fishery population and its long time series of MS data. Algorithm 2 was used for estimating the various summary statistics that are presented in graphical or tabular form to evaluate the performance of $\hat{\gamma}_{u1}$ to $\hat{\gamma}_{w}$.

Algorithm 2:

Set: $\gamma$, $\pi_x$ and $\pi_{\bar{x}}$.

Generate: Using Algorithm 1, randomly draw $s = 5000$ samples of $w_x$ and $w_{\bar{x}}$ i.e. $w_x$ and $w_{\bar{x}}$.

Calculate: $\hat{\gamma}_{u1}(w_x, w_{\bar{x}})$ and $\hat{\gamma}_{w}(w_x, w_{\bar{x}})$.
Calculate:

\[
\widehat{\text{RMSE}}(\hat{y}_i) = \sqrt{\frac{1}{s} \sum_{i=1}^{s} (\hat{y}_i - \gamma)^2} - \sqrt{\frac{1}{s} \sum_{i=1}^{s} (\hat{y}_i - \hat{y}_i)^2}
\]

Approximate the population mean and variance of \(\hat{y}_j\) as:

\[
\E(\hat{y}_j) \approx \frac{1}{s} \sum_{i=1}^{s} \hat{y}_j(w_x[i], w_x[i])
\]

\[
\text{Var}(\hat{y}_j) \approx \sqrt{\frac{1}{s} \sum_{i=1}^{s} (\hat{y}_j(w_x[i], w_x[i]) - \E(\hat{y}_j))^2}
\]

Calculate the estimated variance for estimator \(\hat{y}_j\) by using either the Taylor series approximation or the bootstrap (only used for \(\hat{y}_j\)) method:

\[
\widehat{\sigma}^2_{\hat{y}_j} \approx \frac{1}{s} \sum_{i=1}^{s} \widehat{\sigma}^2_{\hat{y}_j}(w_x[i], w_x[i])
\]

Plot \(\widehat{\sigma}^2_{\hat{y}_j} / \text{Var}(\hat{y}_j)\) for various levels of \(\gamma, \pi_x\) and \(\pi_{\bar{x}}\) to assess the effectiveness of \(\widehat{\sigma}^2_{\hat{y}_j}\) to estimate \(\text{Var}(\hat{y}_j)\).

Construct the Clopper-Pearson (CP) 100(1 - \(\alpha\))% confidence interval lower and upper bound, with a nominal level of \(\alpha = 0.05\), for each of the simulated samples \(i\): \(L_{\hat{y}_i}(\alpha, w_x[i], w_{\bar{x}}[i])\) and \(U_{\hat{y}_i}(\alpha, w_x[i], w_{\bar{x}}[i])\).

Estimate the coverage rate of \(\hat{y}_j\) for \(\E(\hat{y}_j)\) and \(\gamma\):

\[
\widehat{\text{CVR}}_{\hat{y}_j}(\E(\hat{y}_j), \alpha) = \frac{1}{s} \sum_{i=1}^{s} I(\hat{y}_i(\cdot) \leq \E(\hat{y}_j) \leq U_{\hat{y}_j}(\cdot))
\]

\[
\widehat{\text{CVR}}_{\hat{y}_j}(\gamma, \alpha) = \frac{1}{s} \sum_{i=1}^{s} I(\hat{y}_i(\cdot) \leq \gamma \leq U_{\hat{y}_j}(\cdot))
\]

where \(I(x) = 1\) if condition \(x\) is true and \(I(x) = 0\), otherwise.
Plots of $\hat{C}_{VT(y_j)}(E(\hat{y}_j), \alpha)$ for various levels of $\gamma$, $\pi_v$ and $\pi_{\bar{v}}$ are presented to assess the effectiveness of CP to produce confidence intervals with the nominated level of confidence $1 - \alpha$. In tables, the estimate $\hat{\alpha} = 1 - \hat{C}_{VT(y_j)}(E(\hat{y}_j), \alpha)$ may be presented.

Plots depicting the range of $\pi_v$, for which $\hat{C}_{VT(y_j)}(\gamma, \alpha) > 0$, for various levels of $\gamma$ and $\pi_{\bar{v}}$, are presented to assess the effectiveness of CP to include $\gamma$. In determining the range of $\pi_v$, $\hat{C}_{VT(y_j)}(\gamma, \alpha)$ is calculated for all $\pi_v$ to two decimal places such that $\pi_v > \pi_{\bar{v}}$.

The optimality of $\hat{\gamma}_v$ and $\hat{\gamma}_m$ over $\hat{\gamma}_o$ for a given $\gamma$ will be dependent on $\pi_v - \pi_{\bar{v}}$ and hence, most tables and graphics are presented in terms of $\gamma$, $\pi_v$ and $\pi_{\bar{v}}$ so that these required differences between $\pi_v$ and $\pi_{\bar{v}}$ can be described. In practice however, the magnitude of difference between $\pi_v$ and $\pi_{\bar{v}}$ might not be easily assumed and so some tables are presented in terms of ranges on $\gamma$ only to assess the general impact of using one estimator over another, independent of $\pi_v - \pi_{\bar{v}}$. For the k-th range of $\gamma$, described by $\gamma \in \left[\frac{k}{100} - 0.05, \frac{k}{100} + 0.05\right]$, these table entries were constructed by drawing 1000 random samples of $\gamma$, $\pi_v$, and $\pi_{\bar{v}}$ assuming $\gamma \sim U\left(\frac{k}{100} - 0.05, \frac{k}{100} + 0.05\right)$, $\Pi_{\bar{v}} \sim U(0, 1)$ and $\Pi_v \sim U(\pi_{\bar{v}}, 1)$. For each of these randomly selected treatments of $\gamma$, $\pi_v$, and $\pi_{\bar{v}}$, 500 random samples of $w_v$ and $w_{\bar{v}}$ were then drawn using Algorithm 1 and $\hat{y}_{ij}$ calculated for each. Paired with the actual value of $\gamma$ used to generate its random sample, the various summary statistics of $\hat{y}_{ij}$ were then calculated for the half a million random samples and summarized over the selected range of $\gamma$.  
4.7 Results

The range of $\gamma$, $\pi_e$ and $\pi_P$ for which $\hat{\gamma}_m$ and $\hat{\gamma}_e$ are an improvement on $\hat{\gamma}_o$, in terms of reduction in RMSE, are illustrated in Figure 4.1. It is seen that $\pi_e > \pi_P$ is a requirement for the two proposed estimators to be an improvement on $\hat{\gamma}_o$. The $\hat{\gamma}_e$ is generally better than $\hat{\gamma}_o$ over this region for all $\gamma$, except when $\pi_e$ and $\pi_P$ are nearly equal. The $\hat{\gamma}_m$ has a similar domain of optimality to $\hat{\gamma}_e$ for low to medium levels of $\gamma$. Past this point it is seen that as $\gamma$ increases then so does the required difference between $\pi_e$ and $\pi_P$ for $\hat{\gamma}_m$ to outperform $\hat{\gamma}_o$. The region for which $\hat{\gamma}_e$ outperforms $\hat{\gamma}_o$ is largely unchanged over all $\gamma$.

The cost-benefit in using $\hat{\gamma}_e$ and not $\hat{\gamma}_o$ when $\pi_e > \pi_P$ is assessed in Figure 4.2 which illustrates the achieved reduction in RMSE in estimating $\gamma$ by using $\hat{\gamma}_e$ instead of $\hat{\gamma}_o$ for various levels of $\gamma$, $\pi_e$ and $\pi_P$. For small $\gamma$ the increase in RMSE due to using $\hat{\gamma}_e$ when not optimal (error reduction < 0, Figure 4.2) is small relative to the achieved reduction when optimal (error reduction > 0). As $\gamma$ increases, the cost of misusing $\hat{\gamma}_e$ increases although for each given level of $\gamma$ and $\pi_P$ the range of $\pi_e$ for which $\hat{\gamma}_e$ is not optimal, is narrow (as supported by Figure 4.1 which indicates $\hat{\gamma}_e$ is generally better than $\hat{\gamma}_o$ except when $\pi_e$ and $\pi_P$ are nearly equal).

Using $\hat{\gamma}_m$ to estimate $\gamma$ results in reductions in RMSE that are twice as great as those achieved using $\hat{\gamma}_e$ (Figure 4.3). Conversely however, when not optimal, the increase in RMSE is also doubled. For a given level of $\gamma$ and $\pi_P$, the range of $\pi_e$ for which $\hat{\gamma}_m$ is worse than $\hat{\gamma}_o$ is wider than that for $\hat{\gamma}_e$ (as expected given Figure 4.1). When directly comparing $\hat{\gamma}_e$ and $\hat{\gamma}_m$ in terms of RMSE (grey region, Figure 4.4) it is seen that the domain of optimality for $\hat{\gamma}_m$ is largely unchanged to that when compared to $\hat{\gamma}_o$ (red region, Figure 4.4). It is therefore safe to assume that in general, if $\hat{\gamma}_m$ is preferred to $\hat{\gamma}_o$, then it is also preferred to $\hat{\gamma}_e$.

Figures 4.2 and 4.3 are described numerically in Tables 4.2 and 4.3 respectively, and define the average maximum increase (which occurs at $\pi_e = \pi_P$) and decrease to RMSE for using either $\hat{\gamma}_m$ or $\hat{\gamma}_e$ instead of $\hat{\gamma}_o$. These tables also define the minimum required difference between $\pi_e$ and $\pi_P$ for which a particular estimator begins to
outperform \( \hat{\gamma}_o \) in terms of RMSE. For example, referring to the cell relating to row \( \gamma = 0.4 \) and column \( \pi_x = 0.3 \) of Table 4.3, which compares the RMSE of \( \hat{\gamma}_m \) to that of \( \hat{\gamma}_o \), it is seen that maximum average increase in RMSE by using \( \hat{\gamma}_m \) to estimate \( \gamma \) is 0.140 (entry 1) but at best, the RMSE is reduced by 0.166 (entry 3). It is also reported that reductions in RMSE for using \( \hat{\gamma}_m \) instead of \( \hat{\gamma}_o \) commence when the difference between \( \pi_x \) and \( \pi_x \) is 0.120 (entry 2) or \( \pi_x = \pi_x + 0.120 = 0.3 + 0.120 \).

The method used to estimate the variance of each estimator \( \hat{\gamma}_o, \hat{\gamma}_m \) and \( \hat{\gamma}_e \) is assessed by studying Figures 4.5 – 4.8. The first order Taylor series expansion estimates of the variance for \( \hat{\gamma}_o \) and \( \hat{\gamma}_m \) generally lead to a small under-estimate ranging between 0 – 2% of the true value for the considered ranges of \( \gamma, \pi_x, \) and \( \pi_x \), with this bias reducing towards 0 as \( \pi_x, \pi_x \to 1 \). The first order Taylor series expansion is much more biased for estimating the variance of \( \hat{\gamma}_e \) with the range of bias being 0 – 40% above the true value (Figure 4.7). Using the bootstrap method leads to variance estimates of \( \hat{\gamma}_e \) (Figure 4.8) that are comparable in accuracy to those achieved for \( \hat{\gamma}_o \) and \( \hat{\gamma}_m \) using the expansion.

The tendency to under-estimate the variance for each of the three considered estimators of \( \gamma \) will cause the width of the resulting 95% confidence intervals to be narrower than they would be using the true value. Using the conservative CP method for constructing these intervals with these negatively biased variance estimates, still leads to conservative confidence intervals with the observed coverage of the estimator mean, over \( \gamma, \pi_x, \) and \( \pi_x \), actually ranging between 95 – 95.5% (Figures 4.9 – 4.11).

The width of the confidence intervals for each estimator decreases for increasing values of \( \pi_x \) and \( \pi_x \) (Figure 4.12). Generally, the confidence interval widths are smallest for \( \hat{\gamma}_m \). Accompanied by the similarity in coverage rates illustrated for the three estimators (Figures 4.9 – 4.11), this indicates that the confidence intervals for \( \hat{\gamma}_m \) are less affected by sampling variation than the other considered estimators.

The ranges of \( \gamma, \pi_x \) and \( \pi_x \), for which \( CV_{\gamma}(\gamma, \alpha) > 0 \) are presented in Figure 4.13. The range of \( \pi_x \) for which \( CV_{\gamma}(\gamma, \alpha) > 0 \) (black lines) is generally a subset of that
for $\hat{\gamma}_c$ (red lines) for all $\gamma$. This observation is also true for $\hat{\gamma}_m$ (blue lines) when $\gamma \leq 0.3$. For larger values of $\gamma$ the required range of $\pi_r$ for $\text{CVT}_m(\gamma, \alpha) > 0$ narrows and diverges from that required for $\hat{\gamma}_c$ and $\hat{\gamma}_o$ indicating that to achieve some coverage for increasing $\gamma$, the required difference between $\pi_r$ and $\pi_{\bar{r}}$ also increases. Generally, the range of values for $\pi_r$ that lead to $\hat{\gamma}_o$ and $\hat{\gamma}_c$ having some coverage of $\gamma$ is independent of $\gamma$ and $\pi_{\bar{r}}$.

To compare $\hat{\gamma}_o$, $\hat{\gamma}_c$, and $\hat{\gamma}_m$ independent of $\pi_r$ and $\pi_{\bar{r}}$, various statistics were simulated and summarized over different ranges of $\gamma$ (Table 4.4). On average, the RMSE and bias of $\hat{\gamma}_c$ is lower than that of $\hat{\gamma}_o$ for all $\gamma$. The RMSE and bias of $\hat{\gamma}_m$ is much smaller than that for both $\hat{\gamma}_o$ and $\hat{\gamma}_c$ when $\gamma \leq 0.65$. Table 4.4 also confirms that the estimated confidence level of the CP 95% confidence intervals using the approximated variances for each estimator are only marginally conservative and hence, Figure 4.13 (ranges of $\pi_r$ for which $\text{CVT}_m(\gamma, \alpha) > 0$) is not distorted by using highly conservative confidence intervals.

The average bias in $\hat{\gamma}_o$ as a percentage of $\gamma$ is highest for $\gamma \in (0.05,0.15]$ (Table 4.5, $\text{ERROR}(\hat{\gamma}_o) = 131\%$) and lowest for $\gamma \in (0.85,0.95]$ (Table 4.5, $\text{ERROR}(\hat{\gamma}_o) = 5\%$). For the range $\gamma \in (0.05,0.65]$, $\hat{\gamma}_m$ has its highest reduction of this average bias of between 53% and 100% (Table 4.5) and hence, leads to significantly improved estimates of $\gamma$. At between 48% and 61% (Table 4.5), the percentage reduction in bias of $\hat{\gamma}_c$ is lower than that achieved by $\hat{\gamma}_m$ over the same range of $\gamma$ but has the advantage that it does not run the risk of increased bias relative to that in $\hat{\gamma}_o$. For $\gamma > 0.65$, $\hat{\gamma}_m$ increases the relative bias in $\hat{\gamma}_o$ by as much as 333% whilst $\hat{\gamma}_c$ continues to reduce this bias by between 67% and 82% (Table 4.5).
Figure 4.1: Combinations of $\gamma$, $\pi_X$ and $\pi_{\bar{X}}$ for which $\text{RMSE}(\tilde{\gamma}_c) \leq \text{RMSE}(\tilde{\gamma}_o)$ (grey area) and $\text{RMSE}(\tilde{\gamma}_m) \leq \text{RMSE}(\tilde{\gamma}_o)$ (red hatched area) as determined by a comparison of their algebraic constructs. The line of equality (black) is also included for reference. The optimal region for $\tilde{\gamma}_m$ (red) fully to partially overlaps that for $\tilde{\gamma}_e$ (grey).
Figure 4.2: The reduction in RMSE when using $\hat{\gamma}_c$ instead of $\hat{\gamma}_o$ (RMSE($\hat{\gamma}_o$) − RMSE($\hat{\gamma}_c$)) for various levels of $\gamma$, $\pi_x$, and $\pi_{\overline{x}}$ where $\pi_x > \pi_{\overline{x}}$. The RMSE of each estimator has been calculated using $w_x = w\gamma\pi_x$ and $w_{\overline{x}} = w(1 - \gamma)\pi_{\overline{x}}$. Each curve refers to a different level of $\pi_{\overline{x}} \in \{0.1, 0.2, \ldots, 0.9\}$. 

Chapter 4
Figure 4.3: The reduction in RMSE when using $\hat{\gamma}_m$ instead of $\hat{\gamma}_o$ (RMSE($\hat{\gamma}_o$) - RMSE($\hat{\gamma}_m$)) for various levels of $\gamma$, $\pi_x$ and $\pi_{\pi_x}$ where $\pi_x > \pi_{\pi_x}$. The RMSE of each estimator has been calculated using $w_x = w\gamma\pi_x$ and $w_{\pi_x} = w(1 - \gamma)\pi_{\pi_x}$. Each curve refers to a different level of $\pi_{\pi_x} \in \{0.1, 0.2, ..., 0.9\}$. 
Table 4.2: Description of $\hat{\gamma}_c$ compared to $\hat{\gamma}_o$ in terms of 3 entries: 1) the average maximum increase in RMSE between using $\hat{\gamma}_c$ over $\hat{\gamma}_o$ when $\pi_x = \pi_{x_0}$ (the point at which the RMSE of $\hat{\gamma}_o$ is minimized and has a zero bias); 2) the required difference between $\pi_x$ and $\pi_{x_0}$ for $\hat{\gamma}_c$ and $\hat{\gamma}_o$ to have equal RMSE; and 3) the maximum reduction in the RMSE of $\hat{\gamma}_c$ for some value of $\pi_x$. Values have been given to 3 decimal places.

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Table 4.3: Description of $\hat{\gamma}_m$ compared to $\hat{\gamma}_o$ in terms of 3 entries: 1) the average maximum increase in RMSE between using $\hat{\gamma}_m$ over $\hat{\gamma}_o$ when $\pi_x = \pi_y$ (the point at which the RMSE of $\hat{\gamma}_o$ is minimized and has a zero bias); 2) the required difference between $\pi_x$ and $\pi_y$ for $\hat{\gamma}_m$ and $\hat{\gamma}_o$ to have equal RMSE; and 3) the maximum reduction in the RMSE of $\hat{\gamma}_m$ for some value of $\pi_x$. Values have been given to 3 decimal places.

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Figure 4.4: Combinations of $\gamma$, $\pi_x$, and $\pi_{\bar{x}}$ for which $\text{RMSE}(\hat{\gamma}_m) \leq \text{RMSE}(\hat{\gamma}_c)$ (grey area) and $\text{RMSE}(\hat{\gamma}_m) \leq \text{RMSE}(\hat{\gamma}_c)$ (red hatched area) as determined by a comparison of their algebraic constructs. The line of equality (black) is also included for reference. The optimal region for $\hat{\gamma}_m$ (red) partially overlaps that for $\hat{\gamma}_c$ (grey).
Figure 4.5: The average ratio of the “Taylor series” estimated variance of $\hat{\gamma}_o$ calculated using 5000 simulated samples of $w_x$ and $w_o$ for various levels of $\gamma$, $\pi_x$, and $\pi_o$, compared to the actual variance estimated as the variance of the 5000 estimates of $\hat{\gamma}_o$ for that treatment. Each solid line in a panel refers to a different level of $\pi_x \in \{0.1, 0.2, \ldots, 0.9\}$ such that $\pi_x > \pi_o$. 

\[\text{Variance ratio (estimated/actual)}\]
Figure 4.6: The average ratio of the “Taylor series” estimated variance of $\hat{\gamma}_m$ calculated using 5000 simulated samples of $w_x$ and $w_\bar{y}$ for various levels of $\gamma$, $\pi_x$, and $\pi_{\bar{y}}$, compared to the actual variance estimated as the variance of the 5000 estimates of $\hat{\gamma}_m$ for that treatment. Each solid line in a panel refers to a different level of $\pi_x \in \{0.1, 0.2, \ldots, 0.9\}$ such that $\pi_x > \pi_{\bar{y}}$. 
Figure 4.7: The average ratio of the “Taylor series” estimated variance of $\hat{\gamma}_r$ calculated using 5000 simulated samples of $w_x$ and $w_{\pi}$ for various levels of $\gamma$, $\pi_x$ and $\pi_{\pi}$, compared to the actual variance estimated as the variance of the 5000 estimates of $\hat{\gamma}_r$ for that treatment. Each solid line in a panel refers to a different level of $\pi_{\pi} \in \{0.1, 0.2, \ldots, 0.9\}$ such that $\pi_x > \pi_{\pi}$. 
Figure 4.8: The average relativeness of the bootstrapped variance of $\hat{\gamma}_r$ generated for 5000 simulated samples of $w_r$ and $w_{\overline{r}}$ for various levels of $\gamma$, $\pi_r$ and $\pi_{\overline{r}}$, using the bootstrap method with 200 simulations, compared to the actual variance estimated as the variance of the 5000 estimates of $\hat{\gamma}_r$ for that treatment. Each solid line in a panel refers to a different level of $\pi_{\overline{r}} \in \{0.1, 0.2, \ldots, 0.9\}$ such that $\pi_r > \pi_{\overline{r}}$. 
Figure 4.9: The proportion of Clopper-Pearson confidence intervals for $\hat{\gamma}_o$ generated for 5000 simulated samples of $w_o$ and $w_F$ for various levels of $\gamma$, $\pi_F$ and $\pi_{oF}$, that include $E(\hat{\gamma}_o)$ which has been estimated as the average of $\hat{\gamma}_o$ for all the simulated samples for that treatment. The 95% level is also presented (dashed line). Each solid line in a panel refers to a different level of $\pi_F \in \{0.1, 0.2, ...0.9\}$ such that $\pi_x > \pi_F$. 
Figure 4.10: The proportion of Clopper-Pearson confidence intervals for $\hat{\gamma}_m$ generated for a 5000 simulated samples of $w_x$ and $w_y$ for various levels of $\gamma$, $\pi_x$ and $\pi_y$, that include $E(\hat{\gamma}_m)$ which has been estimated as the average of $\hat{\gamma}_m$ for all the simulated samples for that treatment. Each solid line in a panel refers to a different level of $\pi_y \in \{0.1, 0.2, \ldots, 0.9\}$ such that $\pi_x > \pi_y$. The 95% level is also presented (dashed line).
Figure 4.11: The proportion of Clopper-Pearson confidence intervals for $\hat{\gamma}_c$ generated for 5000 simulated samples of $w_x$ and $w_\psi$ for each $\gamma$, $\pi_x$ and $\pi_\psi$, that include $E(\hat{\gamma}_c)$ which has been estimated as the average of $\hat{\gamma}_c$ for all the simulated samples for that treatment. Each solid line in a panel refers to a different level of $\pi_x \in \{0.1, 0.2, \ldots, 0.9\}$ such that $\pi_x > \pi_\psi$. The 95% level is also presented (dashed line).
Figure 4.12: The average width of the Clopper-Pearson confidence intervals for each estimator ($\hat{\gamma}_o$ – black; $\hat{\gamma}_m$ – blue; $\hat{\gamma}_r$ – red) calculated using 5000 simulated samples of $w_r$ and $w_{fr}$ for various levels of $\gamma$, $\pi_r$ and $\pi_{fr}$. Each solid line in a panel refers to a different level of $\pi_{fr} \in \{0.1, 0.2, ..., 0.9\}$ such that $\pi_r > \pi_{fr}$. A confidence level of $\alpha = 0.05$ has been used.
Figure 4.13: The range of $\pi_\gamma$ for which the proportion of Clopper-Pearson confidence intervals for each estimator ($\hat{\gamma}_0$ – black; $\hat{\gamma}_m$ – blue; $\hat{\gamma}_e$ – red) given $\gamma$ and $\pi_\gamma$, is greater than 0. 5000 simulated samples of $\omega_\gamma$ and $\omega_\gamma$ for each $\gamma$, $\pi_\gamma$, and $\pi_\gamma$, have been used for each the estimators. Each solid line in a panel refers to a different level of $\pi_\gamma \in \{0.1, 0.2, \ldots, 0.9\}$ such that $\pi_\gamma > \pi_\gamma$. A line of equality between $\pi_\gamma$ and $\pi_\gamma$ has also been presented (dashed line).
Table 4.4: Various statistics for each estimator \( \hat{\gamma}_o \), \( \hat{\gamma}_m \) and \( \hat{\gamma}_c \), for different ranges of \( \gamma \). For each range of \( \gamma \), 1000 random combinations of \( \gamma \), \( \pi_x \) and \( \pi_y \) are chosen from a uniform distribution such that \( \pi_x > \pi_y \). For each combination of \( \gamma \), \( \pi_x \) and \( \pi_y \), 500 random samples of \( w_x \) and \( w_y \) are drawn and used to estimate the population value for each presented statistic i.e. 1000 x 500 sample points used for calculating each statistic of a particular range of \( \gamma \). The statistics calculated for estimator \( \hat{\gamma}_y \) are: RMSE(\( \hat{\gamma}_y \)), \( b_\gamma(\hat{\gamma}_y) \), \( \hat{\gamma}_y^2 \) and \( (1 - \hat{\alpha}) \) - the proportion of Clopper-Pearson confidence intervals (using the approximate formulas identified for \( \hat{\gamma}_o \) and \( \hat{\gamma}_m \) but bootstrapping for \( \hat{\gamma}_c \)) that include the population mean of estimator \( \hat{\gamma}_y \). The level of confidence \( \alpha = 0.05 \) has been used to construct confidence intervals and \( W=40000 \) and \( w=4000 \) are used for all calculations.

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<th>( b_\gamma(\hat{\gamma}_y) )</th>
<th>( \hat{\gamma}_y^2 )</th>
<th>( 1 - \hat{\alpha} )</th>
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<td>( \hat{\gamma}_o )</td>
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<td>0.0262</td>
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</tr>
<tr>
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<td>( \hat{\gamma}_m )</td>
<td>0.0950</td>
<td>0.0408</td>
<td>0.0074</td>
<td>0.9549</td>
</tr>
<tr>
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<td>0.1403</td>
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<tr>
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<td>0.0083</td>
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<td>0.9569</td>
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Table 4.5: A summary of the results in Table 4.4 - The average relative reduction in the root-mean-square-error \( \text{RelIR}_{\text{RMSE}}(\hat{\gamma}_y|\hat{\gamma}_o) = 100\frac{\text{RMSE}(\hat{\gamma}_o) - \text{RMSE}(\hat{\gamma}_y)}{\text{RMSE}(\hat{\gamma}_o)} \) and absolute bias, \( \text{RelIR}_{\text{Bias}}(\hat{\gamma}_y|\hat{\gamma}_o) = 100\frac{|b_1(\hat{\gamma}_o)| - |b_1(\hat{\gamma}_y)|}{|b_1(\hat{\gamma}_o)|} \), achieved by using \( \hat{\gamma}_m \) or \( \hat{\gamma}_e \) instead of \( \hat{\gamma}_o \) to estimate \( \gamma \), for various ranges of \( \gamma \). The average bias of \( \hat{\gamma}_o \) as a percentage of the average \( \gamma \) \( \left( \text{Error}(\hat{\gamma}_o) = 100\frac{b_1(\hat{\gamma}_o)}{\gamma} \right) \) is also provided to allow assessment of the actual size of error reduction achieved in using \( \hat{\gamma}_m \) or \( \hat{\gamma}_e \) to estimate \( \gamma \) instead of \( \hat{\gamma}_o \).

<table>
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<tr>
<th>( \gamma )</th>
<th>Error(( \hat{\gamma}_o )) (%)</th>
<th>( \hat{\gamma}_y = \hat{\gamma}_m )</th>
<th>( \hat{\gamma}_y = \hat{\gamma}_e )</th>
<th>( \hat{\gamma}_y = \hat{\gamma}_m )</th>
<th>( \hat{\gamma}_y = \hat{\gamma}_e )</th>
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<tr>
<td>(0.35,0.45)</td>
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<td>100</td>
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<tr>
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<tr>
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<td>18</td>
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<tr>
<td>(0.85,0.95)</td>
<td>5</td>
<td>-316</td>
<td>6</td>
<td>-333</td>
<td>82</td>
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</tbody>
</table>

4.8 Discussion

Much research exists that suggests that ‘highly involved’ people are more likely to return a MS form (van Kenhove 2002). The suggestion is that for a particular individual to participate in a survey, the benefit to them must outweigh the cost of doing so. This has previously been evidenced in the recreational western rock lobster fishery where, in comparisons between responses of a MS and a much considered more accurate PDS, it has been shown that people who fished for rock lobster were more likely to return the survey form i.e. \( \pi_r > \pi_T \) where in this case, \( x \) is the characteristic that ‘the person fished’. This has also been seen in other recreational fisheries (Brown & Wilkins 1978; Fisher 1996).

When the probability of fishers and nonfishers returning a MS form are equal, then \( \hat{\gamma}_o \) is an unbiased estimator of the proportion of the surveyed population that fished. Motivated by the observation that fishers are more likely to return a survey form than
Chapter 4

non-fishers, this Chapter constructed \( \hat{\gamma}_m \) and \( \hat{\gamma}_r \) as competitors to the commonly used \( \hat{\gamma}_o \) for estimating the proportion of the surveyed population who have a particular characteristic \( x \) such as “did fish”.

A common criterion for comparing competing estimators, which is a function of the estimators bias and variance, is the root-mean-square-error (RMSE). For a given level of \( \gamma \), the preference of \( \hat{\gamma}_o \), \( \hat{\gamma}_r \) or \( \hat{\gamma}_m \) was dependent on the difference between \( \pi_x \) and \( \pi_y \). Generally, \( \hat{\gamma}_m \) led to significantly greater reductions in RMSE than \( \hat{\gamma}_r \) or \( \hat{\gamma}_o \) but this required greater differences in \( \pi_x \) and \( \pi_y \) for increasing \( \gamma \) (Table 4.3). The required difference between \( \pi_x \) and \( \pi_y \) is much smaller for \( \hat{\gamma}_o \) to outperform \( \hat{\gamma}_o \) (Table 4.2) although the resulting reduction in RMSE compared to those achieved by \( \hat{\gamma}_m \) are smaller.

In practice, determining the preferred estimator based on \( \gamma \) is much more feasible than on both \( \gamma \) and \( \pi_x - \pi_y \). Averaging the achieved reduction in bias over all \( \pi_x \) and \( \pi_y \) (such that \( \pi_x > \pi_y \)), it was seen that on average, \( \hat{\gamma}_m \) outperforms \( \hat{\gamma}_o \) and \( \hat{\gamma}_r \) when \( \gamma \leq 0.65 \). Over this range it is seen that \( \hat{\gamma}_o \) produces an average percentage bias of between 25% and 131% and \( \hat{\gamma}_m \) reduces these by between 53% and 100%. Over the same range, using \( \hat{\gamma}_r \) leads to reductions of between 48% and 61%. Although the percentage reduction in bias of \( \hat{\gamma}_r \) is lower than that achieved by \( \hat{\gamma}_m \) when \( \gamma \leq 0.65 \), it does have the advantage of not running the risk of increasing the bias. For \( \gamma > 0.65 \), \( \hat{\gamma}_m \) increases the average relative bias in \( \hat{\gamma}_o \) by as much as 333% whilst \( \hat{\gamma}_r \) continues to reduce, although the achieved reductions in actual bias in this range are small.

Having estimated \( \gamma \) from a sample of returned survey forms, it is required to understand the variation in this estimate by constructing its confidence intervals. The Clopper-Pearson (CP) confidence intervals are often criticized for being overly conservative (wider than they should be for a given level of confidence). The methods chosen here to estimate the variance for \( \hat{\gamma}_o \), \( \hat{\gamma}_m \) and \( \hat{\gamma}_r \) are seen to underestimate the true variance slightly (< 2%). Using these variance estimates leads to the CP intervals being only marginally conservative.
Given that the variance estimates for each estimator underestimate the true value, no finite-population-correction (FPC) has been applied to account for sampling from a finite population (Scheaffer et al. 1990). The generally accepted heuristic is that a FPC should be applied if more than 5% of the population is being sampled. In this study 10% were sampled. For sample sizes greater than this, the need to apply an FPC needs to be investigated.

To construct \( \hat{\gamma}_e, \pi_v \) and \( \pi_{\bar{v}} \) were assumed to have marginal uniform distributions when calculating their expected values. The uniform distribution is appropriate when all possible response probabilities are equally likely. The \( \hat{\gamma}_e \) could be adjusted to assume these parameters have some other distribution. This could lead to greater reductions in RMSE, than those presented in this research, for situations where those chosen distributions are more appropriate.

Whilst \( \hat{\gamma}_e \) is seen to on average outperform \( \hat{\gamma}_o \) over all \( \gamma \) this does come at the cost of calculating additional parameters for the sample estimate, whereas \( \hat{\gamma}_m \) and \( \hat{\gamma}_o \) only require plugging in values of \( w, w_r, \) and \( w_{\bar{v}} \). Relatively simple formulas also exist to approximate the variance of \( \hat{\gamma}_o \) and \( \hat{\gamma}_m \) whilst \( \hat{\gamma}_e \) relies upon bootstrapping due to the complexity in resulting formulas given the use of these additional parameters that themselves have sampling variation. The additional requirements for calculating \( \hat{\gamma}_e \) and its variance however, are not very demanding on computer time and can be easily coded.

The MS method is employed as a cost effective method for estimating the unknown distribution of some characteristic of the population being sampled. With the return rate of surveys being significantly less than 100%, the returned data is often suspected to suffer from nonresponse bias and many different methods have been employed in the literature to reduce the impact of this post hoc. As a simpler alternative to employing these methods for estimating the proportion of the population that has a particular characteristic such as “did fish”, this Chapter has identified two estimators that lead to significant reductions in RMSE and bias when estimating \( \gamma \). It is proposed that \( \hat{\gamma}_m \) should be used if \( \gamma < 0.65 \) can be safely assumed; else, use \( \hat{\gamma}_e \).
Although the percentage reduction in bias achieved by $\hat{\gamma}_r$ is significantly lower than that by $\hat{\gamma}_m$ for $\gamma \leq 0.65$, it does have the advantage of still being less biased than $\hat{\gamma}_o$ when $\gamma > 0.65$. In Chapter 5 the time series of western rock lobster MS data is used to demonstrate how to choose between $\hat{\gamma}_r$ and $\hat{\gamma}_m$ in a real situation (where $\gamma$ is unknown) so reductions in nonresponse bias can be maximized without the risk of producing estimates that are more biased than when using $\hat{\gamma}_o$. 
Chapter 5

Using an Estimator to Reduce the Impact of Nonresponse Bias on the Total Recreational Catch Estimated from a Mail Survey

Assuming that the phone-diary survey (PDS) is less affected by survey biases such as nonresponse, recall and avidity, Chapter 3 identified a correction factor that standardized the mail survey (MS) catch estimates to those expected using a PDS. In the process of calculating this correction factor it was observed that the probability of returning the MS form was higher for people who fished ($\pi_f$) than those that did not ($\pi_p$) i.e. $\pi_f > \pi_p$. In terms of root-mean-square-error (RMSE), Chapter 4 developed the multinomial ($\gamma_m$) and expectation ($\gamma_e$) estimators as competitors to the traditionally used ratio ($\gamma_o$) estimator for estimating the proportion of a population ($\gamma$) that has some characteristic ($x$), when $\pi_x > \pi_x$. Simulations showed that on average, $\gamma_e$ is preferred to $\gamma_o$ for all $\gamma$. Using $\gamma_m$ leads to significantly less biased estimates than both $\gamma_o$ and $\gamma_e$ when $\gamma \leq 0.65$ but at the cost of being significantly more biased when this is not true. Given $\gamma$ is unknown for real data, questions arise as how to test the suitability for using $\gamma_m$ so greater reductions in nonresponse bias can be achieved without the risk of further biasing the estimate of $\gamma$.

This Chapter demonstrates selecting between $\gamma_e$ and $\gamma_m$ for a real data set. For the series of MS data of the recreational western rock lobster fishery it is identified that using $\gamma_m$ to estimate the proportion of licensees that fished using fishing method $i$ ($\tilde{\gamma}_m^{(i)}$) is ‘best’ for each season $t$. Furthermore, having identified an avidity bias in Chapter 3, the total catch equation is reformulated to include the proportion of fishers for each fishing method that are ‘avid’ ($\tilde{\gamma}_i^{(i)}$) so that $\gamma_m$ can be used to further reduce the bias in the MS catch estimates. Using $\gamma_m$ to estimate $\tilde{p}_{t,i}^{(i)}$ and $\tilde{r}_{t,i}^{(i)}$ results in the MS total catch estimates ($\tilde{T}_{t,i}^{m}$) being significantly closer to those of the PDS ($\tilde{T}_{t,i}^{pds}$) than when using $\gamma_o$. This reduction in bias is largely due to improvements in the
estimated number of fishers \( \hat{L}_{t,v}^{(m)} \) - a function of \( \hat{F}_{t,v}^{(m)} \). The estimates of \( \hat{F}_{t,v}^{(m)} \) were also more similar to their corresponding PDS estimates when using \( \hat{\gamma}_m \) rather than \( \hat{\gamma}_o \) to estimate \( \hat{r}_{t,v}^{(m)} \) but not with the same measure of closeness as between \( \hat{F}_{t,v}^{(m)} \) and \( \hat{L}_{t,v}^{(pds)} \).

Given that \( \hat{\gamma}_m \) requires knowing the number of surveys sent \( (n_i^{(m)}) \), this Chapter continues by demonstrating how to estimate this when unknown. Finally, the estimates \( \hat{L}_{t,v}^{(m)} \), \( \hat{F}_{t,v}^{(m)} \) and \( \hat{r}_{t,v}^{(m)} \), resulting by applying \( \hat{\gamma}_o \) and \( \hat{\gamma}_m \) to the MS data, are compared to the corresponding estimates from the PDS and the results discussed.

5.1 Background

Due to issues such as nonresponse and recall, the total catch estimates made using data collected from a MS method are often suspected of being biased. As was done in Chapter 3 for the recreational western rock lobster fishery, applying a correction factor that calibrates these estimates to those from another survey method that is believed to be more accurate can reduce the total bias in these estimates. If the level of bias in the MS changes over time however, then the reliability of this correction factor is questionable and therefore, running the more reliable, but usually more expensive, survey method intermittently is required to assess its continued usefulness.

Besides identifying a correction factor, other popular methods for adjusting nonresponse bias are the weighting and response propensities methods (discussed in Chapter 2). These methods require identification of all auxiliary data (AD) that significantly explains the nonresponse and the survey variable under study. If this information is not available for both respondents and nonrespondents then these methods can not be applied. Again, if the nonresponse and variable under study are also affected by some unidentified AD that acts in the opposite direction to those that have been employed, then it is even possible to have increased the bias in the otherwise unadjusted estimate. If the use of the estimators identified in Chapter 4 can adequately adjust the total recreational western rock lobster catches for nonresponse
bias, the difficulties with weighting or response propensity methods (through having to identify all significant AD), or the possible inconsistencies and costs of identifying correction factors, can be avoided for this fishery.

5.2 Alternative Total Catch Model

In Chapter 3 the total catch retained by fishing method \( \nu \in \{ \text{pot}, \text{dive} \} \) in season \( t \) was estimated by survey method \( s \in \{ m = \text{Mail}, pds = \text{PDS} \} \) as:

\[
\hat{T}_{t,v,y_p}^{(s)} = \hat{L}_{t,v,y_p}^{(s)} \hat{g}_{t,v,y_p}^{(s)},
\]

where \( \hat{L}_{t,v,y_p}^{(s)} \) is the number of licensees that fished using method \( \nu \) as estimated by using \( \hat{g}_{y_p} \) to estimate \( \hat{p}_{t,v}^{(s)} \) \( (\hat{L}_{t,v,y_p}^{(s)} = \hat{N}_{t,v}^{(s)} \hat{p}_{t,v}^{(s)}), y_p \in \{ e, m, o \} \), and \( \hat{g}_{t,v}^{(s)} \) is the average retained catch per fisher in season \( t \).

To allow for an adjustment in the catch estimates to account for any observed avidity bias, the catch equation needs to be reformulated to include an avidity parameter \( (r_{t,v}^{(m)}) \). The total retained catch for fishing method \( \nu \) in season \( t \) is reformulated as:

\[
\hat{T}_{t,v,y_p,y_o}^{(s)} = \hat{T}_{t,v,y_p}^{(s)} \left( (1 - \hat{r}_{t,v,y_p,y_o}^{(s)} \hat{g}_{t,v,y_p,y_o}^{(s)}) \hat{g}_{t,v}^{(s)} + \hat{r}_{t,v,y_p,y_o}^{(s)} \hat{g}_{t,v}^{(s)} \right),
\]

where:

- \( \hat{g}_{t,v}^{(s)} \) and \( \hat{g}_{t,v,y_p,y_o}^{(s)} \) are the average retained catch per fisher in the season caught using fishing method \( \nu \), as estimated by survey method \( s \), given the person is defined as avid (reported fishing > \( a_{\nu} \) days) or non-avid (reported fishing ≤ \( a_{\nu} \) days), respectively; and
- \( \hat{r}_{t,v,y_p,y_o}^{(s)} \) is the proportion of those people who reported using fishing method \( \nu \) in survey \( s \), that are defined as avid, estimated using \( \hat{g}_{y_o} \) for \( r_{t,v}^{(m)} \) and \( \hat{g}_{y_p} \) for \( p_{t,v}^{(m)} \) (required for estimating the likely number of surveys sent
5.3 Defining ‘Avidity’

The definition of avidity \(a_e\): a fisher is avid if they fished more than \(a_e\) days; and non-avid otherwise) needs to be chosen so the probability of responding for members in the resulting avid group is higher than for the non-avid members. In terms of RMSE, defining \(a_e\) in this way leads to \(\gamma_m\) and \(\gamma_e\) on average, outperforming \(\gamma_o\) for estimating \(r_{t,v}^{(m)}\).

Assuming the proportion of avid fishers reported by the PDS \((\hat{\rho}_{t,v,o,a}^{(pds)})\) are more representative of the true population \(r_{t,v}\) than those from the MS \((\hat{\rho}_{t,v,o,a}^{(m)})\), due to the PDS being generally considered less affected by bias than the MS, then \(\hat{\rho}_{t,v,o,a}^{(m)} > \hat{\rho}_{t,v,o,a}^{(pds)}\) means that avid fisherman are more likely to return the MS form than non-avids i.e. \(\pi_{t,v,a}^{(m)} > \pi_{t,v,a}^{(p)}\).

The change in \(\hat{\gamma}_{t,v,a}\) for different avidity definitions for both potting and diving, along with 95% confidence intervals, are presented in Figures 5.1 and 5.2, respectively. Only avidity definitions in multiples of 5 were considered due to a well known bias in MSs where people tend to round their reported effort and catch to the nearest 5 (Beaman et al. 1997). In Chapter 4 it was identified that on average, \(\hat{\gamma}_m\) outperformed both \(\hat{\gamma}_e\) and \(\hat{\gamma}_o\) when the group having the measured characteristic is more likely to respond than those that do not (e.g. \(\pi_{t,v,e}^{(s)} > \pi_{t,v,o}^{(s)}\) and the true value of the proportion being estimated is \(\leq 0.65\). For potters (Figure 5.1) and divers (Figure 5.2), these conditions \((\pi_{t,v,e}^{(s)} > \pi_{t,v,o}^{(s)}\) and \(\hat{\gamma}_{t,v,o}^{(s)} \leq 0.65\)) hold when the avidity definition is at least 10 and 5 days fished, respectively. Performing a sensitivity analysis on the resulting \(\hat{F}_{t,v}^{(m)}\) for each of the proposed avidity definitions using \(\gamma_m\) to estimate \(\hat{F}_{t,v}^{(m)}\) (Figures 5.3 & 5.4), it is seen that there is a significant reduction in \(\hat{F}_{t,v}^{(m)}\) by applying some avidity definition compared to none at all, but that this reduction is largely
independent of the avidity definition for both potting (Figure 5.3) and diving (Figure 5.4). Given the number of people who reported fishing by potting or diving for the PDS each season (Table 5.1), it was decided to choose $a_{pot} = 10$ days and $a_{dive} = 5$ days so that the difference in number of sample points between the two avidity classes is minimized.

Figure 5.1: Proportion of those people who reported potting, that reported potting (A) > 5 days; (B) > 10 days; (C) > 15 days; and (D) > 20 days, as estimated by the mail (green) and phone-dairy survey (black) for each season. The 95 % bootstrapped percentile confidence intervals using 1000 simulations are also included. A line depicting an avidity rate of 0.65 is also presented (dashed line).
Figure 5.2: Proportion of those people who reported diving, that reported diving (A) > 5 days; (B) > 10 days; and (C) > 15 days, as estimated by the mail (green) and phone-dairy survey (black) for each season. The 95% bootstrapped percentile confidence intervals using 1000 simulations are also included. A line depicting an avidity rate of 0.65 is also presented (dashed line).
Figure 5.3: The average catch per fisher (CPF) calculated for pot fishers each season by applying $\gamma_m$ to the mail survey data to estimate the proportion of those fishers who were avid ($r_{t,v}^{(m)}$) for various definitions of avidity. The mean CPF for all potters is also included (dashed line). The available phone-diary survey estimates, as well as their 95% percentile bootstrap confidence intervals using 1000 simulations, are also included. The CPF using “no avidity” definition is simply the mean of the reported retained catch per potter.
Figure 5.4: The average catch per fisher (CPF) calculated for dive fishers each season estimated by applying $\hat{\gamma}_m$ to the mail survey data to estimate the proportion of those fishers who were avid ($r_{t,v}^{(m)}$) for various definitions of avidity. The mean CPF for all divers is also included (dashed line). The available phone-diary survey estimates, as well as their 95% percentile bootstrap confidence intervals using 1000 simulations, are also included. The CPF using “no avidity” definition is simply the mean of the reported retained catch per diver.
Table 5.1: The number of phone-diary survey participants that reported fishing a range of days by either potting or diving for each season that this survey method was run.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Potting</td>
<td>1-5</td>
<td>54</td>
<td>42</td>
<td>71</td>
<td>50</td>
<td>87</td>
<td>77</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td>41</td>
<td>38</td>
<td>35</td>
<td>30</td>
<td>42</td>
<td>41</td>
<td>13</td>
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<tr>
<td></td>
<td>11-15</td>
<td>34</td>
<td>16</td>
<td>10</td>
<td>9</td>
<td>28</td>
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<td>13</td>
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<td>16-20</td>
<td>12</td>
<td>19</td>
<td>6</td>
<td>5</td>
<td>12</td>
<td>22</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>21-25</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>13</td>
<td>7</td>
</tr>
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<td></td>
<td>&gt;25</td>
<td>43</td>
<td>35</td>
<td>23</td>
<td>15</td>
<td>29</td>
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<td>29</td>
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<td>72</td>
<td>75</td>
<td>57</td>
<td>123</td>
<td>118</td>
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<td></td>
<td>6-10</td>
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<td>16</td>
<td>8</td>
<td>5</td>
<td>12</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

5.4 Which Estimator: \( \hat{\gamma}_m \) or \( \hat{\gamma}_o \)?

The definition of avidity for potting and diving was previously chosen so that \( \hat{\gamma}_m \) would be the most appropriate estimator for estimating \( \bar{\gamma}_t^{(m)} \) for each fishing method \( v \). When \( \pi_{t,v}^{(s)} > \pi_{t,v}^{(s)} \) then \( \hat{\gamma}_o \) is an upper limit on \( p_{t,v} \) (see Chapter 2, p34) and this fact is used to determine the best estimator for estimating \( \bar{\gamma}_t^{(m)} \). Referring to Table 5.2 it is seen that \( \bar{\gamma}_t^{(m)} \) ranges between 0.35 and 0.62, and \( \bar{\gamma}_t^{(m)} \) between 0.22 and 0.36. Given the result in Chapter 4 that \( \hat{\gamma}_m \) is preferred to \( \hat{\gamma}_o \) and \( \hat{\gamma}_o \) when \( p_{t,v} \leq 0.65 \), it is chosen to use \( \hat{\gamma}_m \) for estimating \( \bar{\gamma}_t^{(m)} \) for each fishing method \( v \) and season \( t \) that the MS was run.
Table 5.2: The number of respondents to the recreational western rock lobster fishery mail survey that reported having fished or not by potting and diving for each season. The ratio estimate of the proportion of people who reported fishing using each of these methods ($\hat{p}_{t,v,o}$) is also presented along with the population size ($N_t^{(m)}$) and the number of survey forms sent in each season ($n_t^{(m)}$). The number of surveys sent for seasons 1986/87 – 1994/95 are not known and have been estimated (*) using a procedure presented in Section 5.6.

<table>
<thead>
<tr>
<th>Season ($t$)</th>
<th>$N_t^{(m)}$</th>
<th>$n_t^{(m)}$</th>
<th>Potting ($n$)</th>
<th>Diving ($n$)</th>
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<td></td>
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<td></td>
<td>$n_{t,v}$</td>
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<tr>
<td>1986/87</td>
<td>16484</td>
<td>9269*</td>
<td>2561</td>
<td>1547</td>
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<td>1988/89</td>
<td>22529</td>
<td>14944*</td>
<td>3707</td>
<td>2909</td>
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<tr>
<td>1989/90</td>
<td>23374</td>
<td>1623*</td>
<td>375</td>
<td>315</td>
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<tr>
<td>1990/91</td>
<td>22777</td>
<td>894*</td>
<td>171</td>
<td>226</td>
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<td>1991/92</td>
<td>25907</td>
<td>793*</td>
<td>180</td>
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<td>1992/93</td>
<td>26580</td>
<td>1504*</td>
<td>370</td>
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<td>25079</td>
<td>2474*</td>
<td>577</td>
<td>534</td>
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<td>946</td>
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5.5 Estimating $r_{t,v}^{(m)}$ Using $\hat{\gamma}_m$ and $\hat{\gamma}_o$

Estimating $r_{t,v}^{(m)}$ using $\hat{\gamma}_o$ is independent of the estimate of $\hat{\gamma}_m$ and is only a function of $n_{t,v,a}^{(m)}$ and $n_{t,v,\bar{a}}^{(m)}$ which are calculated directly from the returned MS forms. That is, the ratio estimate of $r_{t,v}^{(m)}$ is:

$$
\hat{r}_{t,v}^{(m)} = \frac{n_{t,v,a}^{(m)}}{n_{t,v,a}^{(m)} + n_{t,v,\bar{a}}^{(m)}},
$$

where for season $t$, $n_{t,v,a}^{(m)}$ and $n_{t,v,\bar{a}}^{(m)}$ are the number of respondents to the MS who reported using fishing method $v \in \{pot, dive\}$ that were avid (fished $> a_v$ days) and non-avid (fished $\leq a_v$ days), respectively.

The $\hat{\gamma}_m$ however, is a function of the total number of survey forms that were sent to sampling units that have the characteristic of interest. In terms of avid fishers for fishing method $v$, this requires determining the number of survey forms that were likely sent to people who did use that fishing method. This requires firstly estimating $\hat{\gamma}_{y_o}$ by some estimator $\hat{\gamma}_{y_o}$ and then multiplying this resulting estimate by $n_{t}^{(m)}$ to get an estimate of the number of surveys likely to have been sent to someone who fished using fishing method $v$ i.e. $n_{t}^{(m)} \hat{r}_{t,v,y_o}^{(m)}$. The estimate of $r_{t,v}^{(m)}$ using $\hat{\gamma}_m$ is then given by:

$$
\hat{r}_{t,v,y_o,m}^{(m)} = \frac{1}{2} \left( \frac{n_{t,v,a}^{(m)}}{n_{t}^{(m)} \hat{\gamma}_{y_o}^{(m)}} + \hat{r}_{t,v,y_o}^{(m)} \right).
$$

5.6 Estimating $n_t$

For seasons 1986/87 – 1994/95, the total number of survey forms sent ($n_t^{(m)}$) was not measured (Figure 5.5). Using $\hat{\gamma}_m$ to estimate $\hat{\gamma}_{y_o}$ and $\hat{r}_{t,v}^{(m)}$ requires knowing $n_t^{(m)}$ and hence, a procedure is required for estimating these for earlier seasons.

Firstly, for seasons that $n_t^{(m)}$ is known, a linear relationship describing total catch $\hat{T}_{t,,m,m}$ (i.e. $\hat{T}_{t,pot,m,m}^{(m)} + \hat{T}_{t,dive,m,m}^{(m)}$) in terms of $\hat{T}_{t,,o,o}^{(m)}$ is fitted (Table 5.3, Figure 5.6).
Figure 5.5: The number of mail survey forms sent \( (n_{t}^{(m)}) \) and returned by recipients who reported having fished \( (n_{t,t}^{(m)}) \) and not \( (n_{t,t}^{(m)}) \) for the annual mail survey of people licensed to fish recreationally for western rock lobster. The number of surveys sent at the end of seasons 1986/87 – 1994/95 is unknown.

Table 5.3: Regressions modeling \( \hat{T}_{t,m}^{(m)} \) in terms of \( \hat{T}_{t-\alpha,0}^{(m)} \). A full model (including an intercept and gradient) and a reduced model where nonsignificant parameters have been removed are presented.

<table>
<thead>
<tr>
<th>Model</th>
<th>R²</th>
<th>Parameter</th>
<th>Estimate</th>
<th>s.e.</th>
<th>t-value</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>0.92</td>
<td>Intercept</td>
<td>-5.23</td>
<td>22.43</td>
<td>-0.23</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gradient</td>
<td>0.6284</td>
<td>0.0399</td>
<td>15.75</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>Reduced</td>
<td>-</td>
<td>Gradient</td>
<td>0.6196</td>
<td>0.0127</td>
<td>48.70</td>
<td>&lt;0.01</td>
</tr>
</tbody>
</table>
To determine estimates of $n_t^{(m)}$, the total catch of potting and diving is combined since $n_t^{(m)}$ must be the same for both fishing methods. With $\hat{T}_t^{(m)}$ known for all seasons, the optimal reduced regression identified in Table 5.3 (and presented in Figure 5.6) is then used to estimate what $\hat{T}_t^{(m)}$ would have likely been if $n_t^{(m)}$ was known ($\hat{T}_t^{(m)}_{est}$). For all possible values of $n^* \in \{n | n_{t,R} \leq n \leq N\}$ (i.e. $n_t^{(m)}$ must be at least the number of respondents to the survey and at most, the size of the population) estimates of $\hat{p}_t^{(m)}$ and $\hat{r}_t^{(m)}$ are made and resulting total catch estimates calculated ($\hat{T}_t^{(m)}(n = n^*)$). The best estimate of $n_t^{(m)}$ ($\hat{n}_t^{(m)}$) is then determined as the value of $n^*$ that minimizes $(\hat{T}_t^{(m)}(n_t^{(m)} = n^*) - \hat{T}_t^{(m)}_{est})^2$.

More optimal searching algorithms were not considered to estimate $n_t^{(m)}$ given the short search time (matter of a few seconds) of the brute force method presented here, for the ranges of $n_t^{(m)}$ in this research.
The success of the presented procedure to estimate $n_t^{(m)}$ is investigated in Figure 5.7. This figure restricts attention to seasons where $n_t^{(m)}$ is known. Estimates of $n_t^{(m)}$ are then attained for each of these seasons by refitting the regression as in Figure 5.6 but for omitting the season being estimated, and then running the brute-force algorithm previously presented for estimating $n_t^{(m)}$. A regression is then fitted between all estimates and their known values of $n_t^{(m)}$ (Figure 5.7). The resulting regression of best fit, with an intercept that is not significantly to 0 ($P = 0.69$) and a gradient that is not different to unity ($P = 0.61$, t-test), suggests that the procedure presented here for estimating $n_t^{(m)}$ is producing mean estimates that are similar to their true $n_t^{(m)}$.

The $\hat{n}_t^{(m)}$ for those seasons where $n_t^{(m)}$ is unknown are presented in Table 5.2 and highlighted by *.

**Figure 5.7:** The $\hat{n}_t^{(m)}$ versus $n_t^{(m)}$ for seasons where $n_t^{(m)}$ is known. Each $\hat{n}_t^{(m)}$ has been calculated by a linear regression estimated with data where the season for which $n_t^{(m)}$ is to be estimated, has been omitted. The line of best fit is also presented ($\hat{n}_t^{(m)} = 316.6 \ (P = 0.69) + 0.90 \ (P < 0.01)n_t^{(m)}$) and is seen to have a gradient that is not significantly different to unity ($P = 0.61$).
5.7 Affect of Applying Nonresponse Correcting Estimator

Using $\hat{\gamma}_o$ or $\hat{\gamma}_m$ to estimate both $\hat{p}_{t,v}^{(m)}$ and $\hat{r}_{t,v}^{(m)}$, a comparison between the resulting MS estimates of $\hat{T}_{t,v}^{(s)}$ (Table 5.4), $\hat{L}_{t,v}^{(s)}$ (Table 5.5) and $\hat{F}_{t,v}^{(s)}$ (Table 5.6) is made to those from the corresponding PDS. The relative difference between $\hat{T}_{t,v,m,m}^{(m)}$ and $\hat{T}_{t,v,o,o}^{(pds)}$ (3 – 128%, Table 5.4) was much smaller than between $\hat{T}_{t,v,o,o}^{(m)}$ and $\hat{T}_{t,v,o,o}^{(pds)}$ (72 – 276%, Table 5.4). These reductions in relative differences between the MS and the PDS estimates is largely due to $\hat{\gamma}_m$ producing estimates $\hat{T}_{t,v}^{(m)}$ for potters (Relative difference: 2 – 19%, Table 5.5) and divers (Relative difference: 3 – 32%, Table 5.5) that were much more comparable to $\hat{L}_{t,v}^{(pds)}$ than when using $\hat{\gamma}_o$ (Relative difference: potters, 35 – 65%; divers, 41 – 85%, Table 5.5). The reduction in relative difference of $\hat{F}_{t,v}^{(s)}$ between the two survey methods was much less for having used $\hat{\gamma}_m$ (Relative difference: potters, -22 – 89%; divers, 46 – 133%, Table 5.6) instead of $\hat{\gamma}_o$ (Relative difference: potters, -7 – 123%; divers, 67 – 176%, Table 5.6).

The results of Tables 5.4 – 5.6 are presented graphically in Figures 5.8 – 5.10 and include all seasons for which the MS has been run. In addition, the estimates of $r_{t,v}^{(s)}$ for each fishing method using $\hat{\gamma}_o$ and $\hat{\gamma}_m$ are presented in Figure 5.11.
Table 5.4: Comparison of the total catch (tonnes) taken using fishing method $v$ in season $t$, estimated by the phone-diary ($\hat{T}_{t,v,o,o}^{(pds)}$) and the mail survey method where the proportion of people who fished ($p_t^{(m)}$) and the proportion of those fishers who were avid ($r_t^{(m)}$), were both estimated using either $\hat{\gamma}_o$ ($\hat{T}_{t,v,o,o}^{(m)}$) or $\hat{\gamma}_m$ ($\hat{T}_{t,v,m,m}^{(m)}$). The relative percentage difference of $\hat{T}_{t,v,g,g}^{(m)}$ to $\hat{T}_{t,v,o,o}^{(pds)}$ for $y \in \{0, m\}$ is also presented i.e.

$$\text{ReID}(\hat{T}_{t,v,g,g}^{(m)} | \hat{T}_{t,v,o,o}^{(pds)}) = 100 \frac{\hat{T}_{t,v,g,g}^{(m)} - \hat{T}_{t,v,o,o}^{(pds)}}{\hat{T}_{t,v,o,o}^{(pds)}}.$$ 

| Method $(v)$ | Season $(t)$ | Estimate | ReID($\hat{T}_{t,v,g,g}^{(m)} | \hat{T}_{t,v,o,o}^{(pds)}$) |
|-------------|-------------|----------|--------------------------------------------------|
| Potting     | 2000/01     | 459.3    | 301.8    | 252.7    | 82 | 19 |
|             | 2001/02     | 399.2    | 249.4    | 162.7    | 145 | 53 |
|             | 2004/05     | 542.9    | 330.2    | 147.8    | 267 | 123 |
|             | 2005/06     | 300.0    | 180.4    | 101.5    | 195 | 78 |
|             | 2006/07     | 245.0    | 158.4    | 85.9     | 185 | 84 |
|             | 2007/08     | 286.3    | 172.8    | 109.2    | 162 | 58 |
|             | 2008/09     | 257.5    | 153.8    | 179.5    | 43 | -14 |
| Diving      | 2000/01     | 162.8    | 108.0    | 65.1     | 150 | 66 |
|             | 2001/02     | 170.8    | 106.7    | 64.4     | 165 | 66 |
|             | 2004/05     | 199.0    | 120.5    | 49.6     | 302 | 143 |
|             | 2005/06     | 102.0    | 62.8     | 29.5     | 245 | 113 |
|             | 2006/07     | 120.0    | 77.9     | 40.7     | 195 | 91 |
|             | 2007/08     | 113.3    | 68.8     | 39.8     | 184 | 73 |
|             | 2008/09     | 150.5    | 89.9     | 57.4     | 162 | 57 |
| Pot + Dive  | 2000/01     | 622.0    | 409.9    | 317.8    | 96 | 29 |
|             | 2001/02     | 570.1    | 356.1    | 227.1    | 151 | 57 |
|             | 2004/05     | 741.9    | 450.6    | 197.4    | 276 | 128 |
|             | 2005/06     | 402.0    | 243.2    | 131.0    | 207 | 86 |
|             | 2006/07     | 365.0    | 236.3    | 126.6    | 188 | 87 |
|             | 2007/08     | 399.7    | 241.6    | 149.0    | 168 | 62 |
|             | 2008/09     | 408.0    | 243.6    | 236.9    | 72 | 3 |
Table 5.5: Comparison of the number of fishers that used fishing method \( \nu \) in season \( t \), estimated by the phone-diary (\( \hat{L}_{t,\nu,o}^{(\text{pds})} \)) and the mail survey method where the proportion of people who fished (\( \hat{p}_{t,\nu}^{(\text{m})} \)) is estimated using \( \hat{\gamma}_{o} \left( \hat{L}_{t,\nu,o}^{(\text{m})} \right) \) or \( \hat{\gamma}_{m} \left( \hat{L}_{t,\nu,m}^{(\text{m})} \right) \). The relative percentage difference of \( \hat{L}_{t,\nu,y}^{(\text{m})} \) to \( \hat{L}_{t,\nu,o}^{(\text{pds})} \) for \( y \in \{ \text{o, m} \} \) is also presented i.e. \( \text{RelD} \left( \hat{L}_{t,\nu,y}^{(\text{m})} \bigg| \hat{L}_{t,\nu,o}^{(\text{pds})} \right) = 100 \frac{\hat{L}_{t,\nu,y}^{(\text{m})} - \hat{L}_{t,\nu,o}^{(\text{pds})}}{\hat{L}_{t,\nu,o}^{(\text{pds})}} \).

| Method  \((\nu)\) | Season \((t)\) | Estimate | \(\hat{L}_{t,\nu,o}^{(\text{m})}\) | \(\hat{L}_{t,\nu,m}^{(\text{m})}\) | \(\hat{L}_{t,\nu,o}^{(\text{pds})}\) | \(\text{RelD} \left( \hat{L}_{t,\nu,y}^{(\text{m})} \bigg| \hat{L}_{t,\nu,o}^{(\text{pds})} \right)\) |
|-----------------|----------------|--------|-----------------|---------------------|-----------------|-----------------|
| Potting         | 2000/01        |        | 21895           | 16531               | 16195           | 35              | 2               |
|                 | 2001/02        |        | 18819           | 13738               | 13120           | 43              | 5               |
|                 | 2004/05        |        | 22282           | 15937               | 13501           | 65              | 18              |
|                 | 2005/06        |        | 17835           | 12663               | 10783           | 65              | 17              |
|                 | 2006/07        |        | 15458           | 11438               | 9624            | 61              | 19              |
|                 | 2007/08        |        | 15677           | 11132               | 10170           | 54              | 9               |
|                 | 2008/09        |        | 15031           | 10597               | 9694            | 55              | 9               |
| Diving          | 2000/01        |        | 12926           | 9759                | 8620            | 50              | 13              |
|                 | 2001/02        |        | 12059           | 8803                | 8567            | 41              | 3               |
|                 | 2004/05        |        | 12149           | 8690                | 8339            | 46              | 4               |
|                 | 2005/06        |        | 10762           | 7641                | 5806            | 85              | 32              |
|                 | 2006/07        |        | 10717           | 7930                | 7470            | 43              | 6               |
|                 | 2007/08        |        | 11373           | 8076                | 7229            | 57              | 12              |
|                 | 2008/09        |        | 11957           | 8429                | 7961            | 50              | 6               |
| Pot + Dive      | 2000/01        |        | 34821           | 26290               | 24815           | 40              | 6               |
|                 | 2001/02        |        | 30878           | 22541               | 21686           | 42              | 4               |
|                 | 2004/05        |        | 34431           | 24627               | 21840           | 58              | 13              |
|                 | 2005/06        |        | 28597           | 20304               | 16589           | 72              | 22              |
|                 | 2006/07        |        | 26175           | 19368               | 17094           | 53              | 13              |
|                 | 2007/08        |        | 27050           | 19208               | 17399           | 55              | 10              |
|                 | 2008/09        |        | 26988           | 19026               | 17655           | 53              | 8               |
Chapter 5

Table 5.6: Comparison of the average retained catch using fishing method \( \nu \) in season \( t \), estimated by the phone-diary \( \hat{F}_{t,v,o,o}^{(pds)} \) and the mail survey method where the proportion of people who fished \((\hat{p}_{t,v}^{(m)})\) and the proportion of those fishers who were avid \((\hat{v}_{t,v}^{(m)})\), were both estimated using either \( \hat{v}_{t,v,0} \) or \( \hat{v}_{t,v,m} \). The relative percentage difference of \( \hat{F}_{t,v,o,o}^{(m)} \) to \( \hat{F}_{t,v,o,o}^{(pds)} \) for \( y \in \{0, m\} \) is also presented i.e. \( \text{ReID}(\hat{F}_{t,v,o,o}^{(m)}|\hat{F}_{t,v,o,o}^{(pds)}) = 100 \frac{\hat{F}_{t,v,o,o}^{(m)} - \hat{F}_{t,v,o,o}^{(pds)}}{\hat{F}_{t,v,o,o}^{(pds)}} \).

| Method \( (\nu) \) | Season \( (t) \) | Estimate | \( \text{ReID}(\hat{F}_{t,v,o,o}^{(m)}|\hat{F}_{t,v,o,o}^{(pds)}) \) |
|----------------|----------------|----------|----------------------------------|
| Potting | 2000/01 | 42.0 | 36.5 | 31.2 | 34 | 17 |
| | 2001/02 | 42.4 | 36.3 | 24.8 | 71 | 46 |
| | 2004/05 | 48.7 | 41.4 | 21.9 | 123 | 89 |
| | 2005/06 | 33.6 | 28.5 | 18.8 | 79 | 51 |
| | 2006/07 | 31.7 | 27.7 | 17.8 | 78 | 55 |
| | 2007/08 | 36.5 | 31.0 | 21.5 | 70 | 45 |
| | 2008/09 | 34.3 | 29.0 | 37.0 | -7 | -22 |
| Diving | 2000/01 | 25.2 | 22.1 | 15.1 | 67 | 46 |
| | 2001/02 | 28.3 | 24.2 | 15.0 | 88 | 61 |
| | 2004/05 | 32.8 | 27.7 | 11.9 | 176 | 133 |
| | 2005/06 | 18.9 | 16.4 | 10.2 | 86 | 62 |
| | 2006/07 | 22.4 | 19.6 | 10.9 | 106 | 80 |
| | 2007/08 | 19.9 | 17.0 | 11.0 | 81 | 55 |
| | 2008/09 | 25.2 | 21.3 | 14.4 | 75 | 48 |
| Pot + Dive | 2000/01 | 35.7 | 31.2 | 25.6 | 39 | 22 |
| | 2001/02 | 36.9 | 31.6 | 20.9 | 76 | 51 |
| | 2004/05 | 43.1 | 36.6 | 18.1 | 138 | 102 |
| | 2005/06 | 28.1 | 24.0 | 15.8 | 78 | 52 |
| | 2006/07 | 27.9 | 24.4 | 14.8 | 88 | 65 |
| | 2007/08 | 29.5 | 25.2 | 17.1 | 73 | 47 |
| | 2008/09 | 30.2 | 25.6 | 26.8 | 13 | -5 |
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Figure 5.8: The estimated total recreational catches (tonnes) for (A) potters, (B) divers, and (C) potters and divers, calculated using either the ratio ($\hat{\gamma}_o$) or multinomial ($\hat{\gamma}_m$) estimator to estimate both the proportion of people who fished ($p_t^{(m)}$) and the proportion of those fishers who were avid ($r_t^{(m)}$), for the mail survey. For seasons 1986/87 – 1994/95, the number of survey forms sent ($n_t^{(m)}$) has been estimated to allow estimates using $\hat{\gamma}_m$. The phone-diary survey estimates are also included. The 95% bootstrapped percentile confidence intervals using 1000 simulations, that also include the sampling variation due to estimating $n_t^{(m)}$ when unknown, are included for all estimates.
Figure 5.9: The estimated number of fishers who (A) potted and (B) dived, calculated using either the ratio ($\hat{\gamma}_r$) or multinomial ($\hat{\gamma}_m$) estimator to estimate the proportion of people who fished ($\hat{\rho}_{t,v}^{(m)}$), for the mail survey. For seasons 1986/87 – 1994/95, the number of survey forms sent ($n_t^{(m)}$) has been estimated to allow estimates using $\hat{\gamma}_m$. The phone-diary survey estimates are also included. The 95% bootstrapped percentile confidence intervals using 1000 simulations, that also include the sampling variation due to estimating $n_t^{(m)}$ when unknown, are included for all estimates.
Figure 5.10: The estimated number of lobsters retained per fisher by (A) potting and (B) diving, resulting by using either the ratio ($\hat{\gamma}_p$) or multinomial ($\hat{\gamma}_m$) estimator to estimate both the proportion of people who fished ($\hat{p}_{f,c}^{(m)}$) and the proportion of those fishers who were avid ($t_{t,c}^{(m)}$), for the mail survey. For seasons 1986/87 – 1994/95, the number of survey forms sent ($n_t^{(m)}$) has been estimated to allow estimates using $\hat{\gamma}_m$. The phone-diary survey estimates are also included. The 95 % bootstrapped percentile confidence intervals using 1000 simulations, that also include the sampling variation due to estimating $n_t^{(m)}$ when unknown, are included for all estimates.
Figure 5.11: The estimated proportion of those fishers who are avid for (A) potters and (B) divers, resulting by using either the ratio ($\hat{\gamma}_o$) or multinomial ($\hat{\gamma}_m$) estimator to estimate both the proportion of people who fished ($p_{i,v}^{(m)}$) and the proportion of those fishers who were avid ($r_{i,v}^{(m)}$). For seasons 1986/87 – 1994/95, the number of survey forms sent ($n_{i,v}^{(m)}$) has been estimated to allow estimates using $\hat{\gamma}_m$. The phone-diary survey catch estimates are also included. The 95% bootstrapped percentile confidence intervals using 1000 simulations, that also include the sampling variation due to estimating $n_{i,v}^{(m)}$ when unknown, are included for all estimates.
5.8 Discussion

The difference between the MS and PDS total catch estimates for potting and diving were more than halved for all seasons when using \( \hat{\gamma}_m \) and not \( \hat{\gamma}_o \) to estimate the proportion of people who fished (\( \hat{p}_{t,v} \)) and the proportion of those who were avid (\( \hat{r}_{t,v} \)) from the MS data. This reduction in difference was largely achieved through \( \hat{\gamma}_m \) producing less biased estimates of number of fishers (\( \hat{L}_{t,v}^{(m)} \) - a function of \( \hat{p}_{t,v} \)). Some reduction of the bias in MS total catch estimates was due to improved estimates of average retained catch per fisher (\( \hat{\mathcal{F}}_{t,v}^{(m)} \) - a function of \( \hat{r}_{t,v} \)), but the achieved reduction in bias of \( \hat{L}_{t,v}^{(m)} \) was much less than that for \( \hat{L}_{t,v}^{(m)} \).

Whilst similar, the estimates of \( L_{t,v}^{(m)} \) using \( \hat{\gamma}_m \) were always higher than \( \hat{\mathcal{F}}_{t,v}^{(pds)} \). This observation may be the result of not having removed all the nonresponse bias from the MS estimates and/or it may be due to a lack of coverage for the PDS. Whilst the PDS made use of all licence holders that existed during the course of the season to determine the population size, it only surveyed people who were licensed to fish for lobster before the commencement of each season. If new licence holders that enter the fishery during the course of the season are more likely to fish for lobster than those who were licensed before the season, due to for example long term licence holders continuing to hold their licence although they do not plan to fish now but do in future seasons, the PDS would underestimate \( \hat{p}_{t,v}^{(pds)} \) and hence, \( \hat{L}_{t,v}^{(pds)} \).

When fishers are more likely to return a MS than non-fishers, \( \hat{\gamma}_o \) represents an upper limit on \( p_{t,v} \). For both potting and diving, \( \hat{p}_{t,v,0}^{(m)} \leq 0.65 \) for all seasons of the recreational western rock lobster fishery that the MS was run. Given \( \hat{\gamma}_m \) leads to greater reductions in nonresponse bias than \( \hat{\gamma}_e \) when \( \gamma \leq 0.65 \), \( \hat{\gamma}_m \) was determined appropriate for estimating \( p_{t,v} \) from the MS.

The value of \( r_{t,v} \) is dependent on the definition of avidity (fished > \( a_v \) days). To define \( a_v \) for the western rock lobster fishery, the proportion of fishers that reported fishing more than a set number of days were compared between the PDS and MS. So that maximum reductions of avidity bias in \( \hat{\mathcal{F}}_{t,v}^{(m)} \) could be achieved, these
comparisons were used to define $a_v$ so that $\hat{\gamma}_m$ was most appropriate for estimating $r_{t,v}$ from the MS. The resulting avidity definition for potters and divers was 10 and 5 days, respectively. The failure of $\hat{\gamma}_m$ to produce estimates of $r_{t,v}$ that resulted in $\hat{F}_{t,v}^{(m)}$ being more similar to $\hat{F}_{t,v}^{(pds)}$ could be partly due to recall bias.

The MS form is sent to lobster fishers at the end of the season without notice unlike the PDS that gives notice to fishers before the start of the season that they will be called regularly to give their catch and fishing information. The different level of success for $\hat{\gamma}_m$ to reduce $\hat{F}_{t,v}^{(m)}$ and $\hat{F}_{t,v}^{(m)}$ to those estimates of the PDS might then be explained by respondents of the MS being able to accurately recall whether or not they used a particular fishing method more so than the number of days they fished.

Connelly & Brown (1995) cited literature that supports the belief that generally, people can use episode enumeration for “infrequent” events, but more likely use some rule-based enumeration for more “mundane, frequent” events. For the recreational rock lobster MS, this might suggest that fishers multiply their average number of lobsters retained per trip by their recall of how many days they fished in the season. In Chapter 3 it was observed that for various ranges of days fished for both potting and diving, $\hat{F}_{t,v}^{(m)}$ and $\hat{F}_{t,v}^{(pds)}$ were similar. This does not mean that the reported number of days fished however, is accurate. If, as in the study of Connelly & Brown (1995), the recall of number of days fished is positively biased, then at least part of the difference between $\hat{r}_{t,v}^{(m)}$ and $\hat{r}_{t,v}^{(pds)}$ is due to misreporting, with some non-avid fishers being misidentified as avid fisherman. The observation that $\hat{F}_{t,v}^{(m)}$ and $\hat{F}_{t,v}^{(pds)}$ are similar for various ranges of days fished may be the result of the previous observation (see Chapter 3) that respondents to the MS have a good ability to report daily catch rates that are well correlated with a known index of lobster abundance.

Although assumed less biased than the MS, it may be possible that the difference between $\hat{r}_{t,v}^{(m)}$ and $\hat{r}_{t,v}^{(pds)}$ may also be partly due to a bias in $\hat{r}_{t,v}^{(pds)}$. PDSs can suffer from ‘soft-refusals’ where for example, a willing participant may occasionally say they did not fish even though they had, to cut the interview short. This sort of bias
would result in the PDS underestimating the average number of days fished and hence, $r_{t,x}^{(pds)}$ to underestimate $r_{t,x}$.

The $\hat{\gamma}_m$ was constructed in Chapter 4 to reduce the impact of the nonresponse bias as identified in Chapter 3. A recall bias (Beaman et al. 2005) is also recognized as an issue with self-administered surveys such as the MS method. The annual MS of recreational rock lobster fishers occurs at the end of the 7.5 month season where most of the fishing occurs in the first 2 months of the season (Melville-Smith et al. 2000). The large amount of time between the peak period of fishing and the survey form being sent is likely to impact on a person’s ability to accurately report the total number of lobsters that they caught and retained. This recall bias is less likely to interfere with a person’s ability to accurately remember whether or not they did fish for rock lobster. Given that $\hat{\gamma}_m$ was designed to reduce the impact of nonresponse and not recall bias, it was not surprising that $\hat{\gamma}_m$ produced estimates of $L_{t,x}$ but not $F_{t,x}$, that were closely agreeable to those from the PDS. The modelling of the recall bias in reported days fished for the recreational western rock lobster MS can not be completed using currently available data and will require future research that is beyond the scope of this thesis.

Under the assumption that $\pi_v > \pi_\overline{v}$, using $\hat{\gamma}_v$ will produce on average, less biased estimates of $\gamma$ than $\hat{\gamma}_o$ for all $\gamma$. Using $\hat{\gamma}_m$ will result in significantly less biased estimates than both $\hat{\gamma}_o$ and $\hat{\gamma}_r$ when $\gamma < 0.65$ but at the cost of being significantly more biased when this is not true. To minimize without the risk of actually increasing the affects of nonresponse bias, this Chapter demonstrated how to determine if $\hat{\gamma}_m$ is appropriate for a real data set where $\gamma$ is unknown. For the MS of recreational western rock lobster fishers it was seen that $\hat{\gamma}_o$ could be used to make this decision for estimating $p_{t,x}$.

Unlike for $p_{t,x}$ the choice of estimator for $r_{t,x}$, which firstly required defining $a_{w}$, was predetermined by comparing MS estimates to those of the more reliable PDS. Access to a secondary, more reliable data set may not exist for other MSs in other fisheries. In such cases, $a_{w}$ needs to be determined so that the probability of returning the MS form for avid fishers ($\pi_{\alpha}$) is greater than that for non-avid ($\pi_{\overline{\alpha}}$) i.e. $\pi_{\alpha} > \pi_{\overline{\alpha}}$. A
possible method for assessing if $\pi_a > \pi_\alpha$ is true, or more generally $\pi_x > \pi_\tau$, when a more reliable data source such as a PDS is not available, is discussed later in Chapter 7. Having determined $\theta_v$ such that $\pi_a > \pi_\alpha$, then similarly as for $p_{t,v}$, $\widehat{\gamma}_o$ can be used to assist in choosing between using $\widehat{\gamma}_r$ and $\widehat{\gamma}_m$ to estimate $r_{t,v}$.

In Chapter 6 the total catch estimates for potting and diving made using $\widehat{\gamma}_m$ and $\widehat{\gamma}_o$ to estimate $p_{t,v}^{(m)}$ and $i_{t,v}^{(m)}$, are modelled. Any differences between these models will be identified and their implications for management, discussed.
Chapter 6

Modelling the Western Rock Lobster Recreational Catch

Comparing the annual mail survey (MS) results of the western rock lobster recreational fishery to those from a more reliable phone-diary survey (PDS), Chapter 3 identified that the probability of returning the MS form was higher for people who fished ($\pi_f$) than those that did not ($\pi_{f,\text{not}}$) i.e. $\pi_f > \pi_{f,\text{not}}$. An avidity bias was also identified where fishers who were ‘avid’ were more likely to return the survey form than the less ‘avid’ fishers. These observed biases of highly ‘interested’ or ‘involved’ people being more likely to return a MS form have also been observed in other studies. In Chapter 4 less biased estimators than the traditionally used ratio estimator ($\hat{\gamma}_o$) were identified for estimating the proportion of the population ($\gamma$) with characteristic $x$ when $\pi_x > \pi_{\overline{x}}$. It was shown that on average, the expectation estimator ($\hat{\gamma}_e$) produces less biased estimates than $\hat{\gamma}_o$ for all $\gamma$. If it can be assumed $\gamma \leq 0.65$, then the multinomial estimator ($\hat{\gamma}_m$) produces even more accurate estimates than $\hat{\gamma}_e$ but at the risk of being more biased than both $\hat{\gamma}_e$ and $\hat{\gamma}_o$ if $\gamma \leq 0.65$ is not true.

To reduce the risk of actually further biasing estimates, Chapter 5 demonstrated for the recreational western rock lobster fishery MS how to choose between $\hat{\gamma}_e$ and $\hat{\gamma}_m$ for estimating the proportion of licensees that fished ($p_{t,v}$) and the proportion of these fishers that were avid ($r_{t,v}$). The choice of estimator for $p_{t,v}$ was determined by studying the estimates of $p_{t,v}$ using $\hat{\gamma}_o$, applied to the MS data. The choice of estimator for $r_{t,v}$ however, which firstly required defining avidity (avid fished $> a_v$ days), was predetermined by comparing various estimates of the MS and PDS and selecting $a_v$ so $\hat{\gamma}_m$ was most appropriate for estimating $r_{t,v}$.

Having determined $\hat{\gamma}_m$ as most appropriate for estimating both $p_{t,v}$ and $r_{t,v}$, the resulting estimates of number of fishers ($\hat{\ell}_{t,v}^{(s)}$ - a function of $\hat{p}_{t,v}$) and average retained catch per fisher ($\hat{F}_{t,v}^{(s)}$, a function of $\hat{r}_{t,v}$) were then compared to those from
the PDS. Estimates \( \hat{L}_{t,v}^{(m)} \) were much more similar to \( \hat{L}_{t,v}^{(pds)} \) when using \( \hat{\gamma}_m \) (and not \( \hat{\gamma}_o \)) to estimate \( \hat{p}_{t,v}^{(m)} \). The estimates of \( \hat{L}_{t,v}^{(m)} \) were also more similar to their corresponding PDS estimates but not with the same measure of closeness as between \( \hat{L}_{t,v}^{(m)} \) and \( \hat{L}_{t,v}^{(pds)} \). It was suggested that a person being unable to accurately recall their number of fishing days may have led to some people being misclassified as avid and may explain at least some of the difference in these results. The total catch estimates of the MS are much closer to those of the PDS when using \( \hat{\gamma}_m \) rather than \( \hat{\gamma}_o \) to estimate both \( \hat{p}_{t,v}^{(m)} \) and \( \hat{r}_{t,v}^{(m)} \).

To implement effective management changes to control the total catch take, managers are aided by understanding the drivers of that catch. This Chapter models \( \hat{L}_{t,v,y,yp}^{(m)} \) and \( \hat{F}_{t,v,y,yp,ya}^{(m)} \) for each fishing method \( v \) to assess whether the interpretation of the resulting models is affected by the estimator, \( \hat{\gamma}_o \) or \( \hat{\gamma}_m \), used to estimate \( \hat{p}_{t,v}^{(m)} \) and \( \hat{r}_{t,v}^{(m)} \) i.e. \( y_o, y_a = 0 \) or \( y_o, y_a = m \). With exception to a scaling factor for \( \hat{F}_{t,v,yp,yo}^{(m)} \), it is seen that the resulting models are unaffected by this choice. The \( \hat{\gamma}_m \) was constructed to reduce the impact of nonresponse bias on the estimate of parameters such as \( \hat{p}_{t,v}^{(m)} \) and \( \hat{r}_{t,v}^{(m)} \). If future work to adjust for recall bias, suspected of affecting the estimate of \( \hat{r}_{t,v}^{(m)} \) and hence \( \hat{F}_{t,v,yp,yo}^{(m)} \), leads to even further agreement between the total catch estimates of the MS and PDS, then our understanding of the drivers influencing the catch take from the western rock lobster fishery by recreational fishers is unlikely to change.

This Chapter continues by identifying and fitting optimal models describing \( \hat{F}_{t,v,yp}^{(m)} \) and \( \hat{F}_{t,v,yp,yo}^{(m)} \), resulting from using either \( \hat{\gamma}_o \) or \( \hat{\gamma}_m \) to estimate both \( \hat{p}_{t,v}^{(m)} \) and \( \hat{r}_{t,v}^{(m)} \), for both potting and diving. Within fishing method, the fitted models are then compared between estimator in terms of their model fits and forecasting accuracy. Finally, any management implications of the identified models are discussed.
6.1 Background

Fishing for western rock lobster is a popular recreational activity where most of the catch is taken by either potting or diving. People have long been required to hold a rock lobster specific licence (RL) to be able to participate in this fishery. In 1992 an ‘umbrella’ licence was also introduced that not only allowed people to fish for rock lobster but at a discounted rate to purchasing all licences separately, in four other recreational fisheries. Since the 1986/87 season the number of people licensed to fish recreationally for rock lobster has grown steadily from 16500 to as many as 47000 in 2003/04 and in 2010/11 was 37900.

Western rock lobsters first appear on the inshore reefs of the West Australian coast as puerulus one year after they are hatched from eggs in the deeper waters off the coast. The abundance of puerulus settling on the inshore reefs each year has long been measured at various sites along the coast (see Section 2.2 in Chapter 2 for further details). Using the time series of puerulus indices for these various collector sites 3 and 4 years earlier, the interannually fluctuating commercial catches of western rock lobster up and down the coast are reliably explained (Morgan et al. 1982; Phillips 1986; Caputi et al. 1995). These relationships generally indicate that with exception to a smaller proportion of fast growing 3 year olds, the majority of western rock lobsters enter the fishery as legally sized animals at approximately 4 years post settlement. Melville-Smith et al. (2001) and (2004) has shown that puerulus settlement at Alkimos (refer to Figure 2.1 in Chapter 2), a location just north of the Perth metropolitan area where the majority of recreational fishing occurs, is well correlated with the recreational lobster catches 3 and 4 years later.

6.2 Modeling Total Catch Estimates

The estimated total catch retained for season $t$ for fishing method $v$ as derived using the MS data is:

$$\hat{T}_{t,v;\theta_0,\theta_0}^{(m)} = \hat{L}_{t,v;\theta_0,\theta_0}^{(m)} F_{t,v;\theta_0,\theta_0}^{(m)} + \varepsilon_t,$$
where:

- \( \hat{L}^{(m)}_{t,v,y_0} = \lambda^{(m)}_{t,v,y_0} \) is the estimated number of licensees that fished;
- \( \hat{F}^{(m)}_{t,v,y_0,y_a} = (1 - \hat{p}^{(m)}_{t,v,y_0,y_a}) \hat{E}^{(m)}_{t,v,y_0} + \hat{p}^{(m)}_{t,v,y_0,y_a} \hat{E}^{(m)}_{t,v,y_0} \) is the estimated retained catch per fisher as a function of the proportion of fishers that are avid i.e. \( \hat{r}^{(m)}_{t,v,y_0,y_a} = \hat{\gamma}_y \left( w = n^{(s)}_{t} \hat{p}^{(m)}_{t,v,y_0}, w_x = n^{(m)}_{t,v,y_0}, w_x = n^{(m)}_{t,v,y_0} \right) \), and
- \( \epsilon_t \sim N(0, \sigma^2_t) \).

The \( \hat{L}^{(m)}_{t,v,y_0} \) and \( \hat{F}^{(m)}_{t,v,y_0,y_a} \) will be calculated using either \( \hat{\gamma}_o \) and \( \hat{\gamma}_m \) to estimate each of the input parameters \( \hat{p}^{(m)}_{t,v} \) and \( \hat{r}^{(m)}_{t,v} \) i.e. \( y_o = y_a \).

The \( \hat{L}^{(m)}_{t,v,y_0} \) and \( \hat{F}^{(m)}_{t,v,y_0,y_a} \) will be modelled separately by a multiple regression and a time series model respectively, with \( \ln(P_{t-3}) \) and/or \( \ln(P_{t-4}) \) the log of the puerulus settlement index at Alkimos 3 and 4 years prior respectively, being considered as regression variables. Only a subset of the available data series (1986/87 – 2002/03) is used to identify the optimal models. The remaining data (2003/04 – 2010/11) will be used to compare the ability of these models under \( \hat{\gamma}_o \) or \( \hat{\gamma}_m \), to produce accurate 1-step ahead forecasts where the prediction for season \( t \) is made using the model fitted with data up until season \( t - 1 \).

The Akaike Information Criterion (AIC) (Sakamoto et al. 1986) is a well used criterion for choosing between competing models. Under this criterion, the optimal model is that which minimizes \( AIC = -2\ln(Lik) + 2n_{par} \) where \( Lik \) is the likelihood of the model and \( n_{par} \) is the number of estimated parameters. Given the relatively small number of data points used to identify the optimal models in this study (only 17), the AIC with correction, \( AICc = AIC + 2n_{par}(n_{par} + 1)/(n_{obs} - n_{par} - 1) \) (Hurvich & Tsai 1989) is considered more appropriate and is used to determine which regressors should be included in the models of \( \hat{L}^{(m)}_{t,v,y_0} \) and \( \hat{F}^{(m)}_{t,v,y_0,y_a} \) as well as the order of the time series model for \( \hat{L}^{(m)}_{t,v,y_0} \).
Chapter 6

The mean-absolute-relative-bias (MARB):

\[
|\tilde{R}(z_2|z_1)| = \frac{1}{j} \sum_{i=1}^{j} \frac{|z_{2,i} - z_{1,i}|}{z_{1,i}}
\]

and mean-relative-bias (MRB):

\[
\tilde{R}(z_2|z_1) = \frac{1}{j} \sum_{i=1}^{j} \frac{z_{2,i} - z_{1,i}}{z_{1,i}}
\]

measure the difference between two series of data \{\{z_2\}\} and \{\{z_1\}\}, relative to \{\{z_1\}\}. The smaller \(\tilde{R}(z_2|z_1)\) and \(\tilde{R}(z_2|z_1)\), then the closer is \(z_2\) to \(z_1\).

Having identified the optimal models for \(\hat{L}_{t,v,y_p}\) and \(\hat{F}_{t,v,y_p,y_a}^{(m)}\), MARB and MRB are used to measure the difference between the resulting model estimates and 1-step ahead forecasts to the observed data to assess if models with better fits and forecasting accuracy result from using one estimator over another (\(\gamma_o\) or \(\gamma_m\)) in estimating \(p_{t,v}\) and \(r_{t,v}\).

The exponential regression model of the form \(\hat{F}_{t,v,y_p,y_a}^{(m)} = e^{\theta_0 + \theta_1 \ln P_{t-3} + \theta_2 \ln P_{t-4}} + \varepsilon_t\) is used to model the average retained catch per fisher for each season \(t\). This model is an adjustment (adjusted to account for the catch of a single fisher and not all fishers) of the Ricker stock recruitment curve (Hilborn & Walters 1992) that describes the number of juveniles that eventually recruit to the fishery as legal sized animals in the presence of a density dependent mortality that limits the carrying capacity of the fishery. This model has been considered appropriate for describing the catches of the western rock lobster fishery (Philips et al. 2003). For each model variation that includes or excludes each of the puerulus indices used as regression variables, the Ljung & Box (1978) test statistic, which tests ‘overall’ serial correlation for a given number of lags (as opposed to testing for just one lag), was used to test for any serial correlation existing in the resulting residuals. No serial correlation means that a time series model does not need to be investigated.
A process \( \{X_t\} \) is said to follow an ARMA\((p,q)\) process if \( \{X_t\} \) is weakly stationary (\( \mu_{X_t} = \mu, \forall t \) and \( \sigma_{X_t,X_s} = \gamma_{t-s} \) is independent of \( s \) & \( t \)) and if \( \forall t: \)
\[
X_t = \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}, \quad \text{where} \quad \{\epsilon_t\} \sim N(0, \sigma^2) \]
and \( \sigma_{\epsilon_t,\epsilon_s}, s \neq t \).

The methodology of Box & Jenkins (1976) was used to guide which ARMA models should be considered to model \( \hat{L}_{t,v,y_p}^{(m)} \). This required visual inspection of the autocorrelation (ACF) and partial autocorrelation (PACF) plots of \( \hat{L}_{t,v,y_p}^{(m)} \) to identify what orders of AR and MA processes should be considered before estimating the model parameters of each. ARMA models are intuitively reasonable for modelling since it should be expected that the scale of licence usage which can change markedly over time, should be closely related to the previous season due to factors such as population growth and the general accessibility of that population to the fishery (e.g. proximity to fishing locations, ownership of boats) that are not explained by the available regressors.

To produce reliable forecasts, a time series model needs to be stationary and invertible (Hamilton 1993). A time series is stationary if all roots of the characteristic polynomial for the AR part of the model \( (\phi_{q_1}(B)) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_{q_1} B^{q_1} \) are significantly outside of the unit circle i.e. \( |B| > 1 \) for all \( B \) such that \( \phi_{q_1}(B) = 0 \). Similarly, a time series is invertible if all roots of the characteristic polynomial for the MA part of the model \( (\theta_{q_3}(B)) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_{q_3} B^{q_3} \) are significantly outside of the unit circle.

When a series is nonstationary, differencing can be used to make the series otherwise and leads to ARIMA\((q_1,d,q_2)\) where \( d \) is the number of times the series is differenced before being modelled by ARMA\((q_1,q_2)\). Log transforming the time series can be used as an alternative to differencing and is suitable for modelling count data such as \( \hat{L}_{t,v,y_p}^{(m)} \). The optimal model for \( \ln \hat{L}_{t,v,y_p}^{(m)} \) is back-transformed to obtain the optimal model for \( \hat{L}_{t,v,y_p}^{(m)} \).
Various ARMA($q_1,q_2$) for low levels of $q_1$ and $q_2$ were considered for modelling $\ln \hat{L}_{t,v,y_p}^{(m)}$. Having determined the optimal ARMA($q_1,q_2$) model, the significance of including $\ln P_{t-3}$ and/or $\ln P_{t-4}$ as regression variables was investigated using AICc. Using AICc to compare models requires using the same response data series for each model. For a series of size $x_1, x_2, \ldots, x_n$, this requires modelling only $x_{\max(q_1)+\max(q_2)+1}, \ldots, x_n$ to allow for the loss of data points when estimating additional parameters.

The model fitting of ARMA was completed in S-Plus (version 8.0.4, Insightful Corp.) using \texttt{arima.mle} which uses a maximum likelihood method to estimate the parameters. This function requires that the series being modelled has a mean of zero(0) to allow the resulting forecast estimates to be reflective of the series. Thus, $\ln \hat{L}_{t,v,y_p}^{(m)}$ was modelled with the mean of the data used for model estimation removed and similarly for any included regression variables.

Having identified models for $\hat{L}_{t,v,y_p}^{(m)}$ and $\hat{F}_{t,v,y_p}^{(m)}$, the variance of $\hat{F}_{t,v,y_p,y_a}^{(m)}$ is estimated as

$$\hat{\sigma}^2_{\hat{F}_{t,v,y_p,y_a}^{(m)}} = \left( \hat{L}_{t,v,y_p}^{(m)} \right)^2 \left( \frac{\hat{\sigma}^2_{\hat{L}_{t,v,y_p}^{(m)}}}{L^2} + \frac{\hat{\sigma}^2_{\hat{F}_{t,v,y_p}^{(m)}}}{F^2} + 2 \frac{\hat{\sigma}_{\hat{L}_{t,v,y_p}^{(m)}} \hat{\sigma}_{\hat{F}_{t,v,y_p}^{(m)}}}{L F} \right)$$

(Kendall & Stuart 1963) where $\hat{L} = \hat{L}_{t,v,y_p}^{(m)}$, $\hat{F} = \hat{F}_{t,v,y_p}^{(m)}$ and independence is assumed between $\hat{L}_{t,v,y_p}^{(m)}$ and $\hat{F}_{t,v,y_p}^{(m)}$ i.e. $\hat{\sigma}_{\hat{L},\hat{F}} = 0$. The 95% confidence intervals for $\hat{F}_{t,v,y_p}^{(m)}$, $\hat{F}_{t,v,y_p}^{(m)}$, and $\hat{F}_{t,v,y_p,y_a}^{(m)}$ are approximated using Newton’s linearization method (Bates & Watts 1988).

\textbf{Modelling catch per fisher $\hat{F}_{t,v,y_p,y_a}^{(m)}$}

The predictors to include in the models of $\hat{F}_{t,v,y_p,y_a}^{(m)}$ for each fishing method and estimator $y_p$, is assessed by comparing the AICc for the various models constructed using the data of seasons 1986/87 – 2002/03 only (Table 6.1). Using $\hat{\gamma}_o$ or $\hat{\gamma}_m$ to estimate $\hat{P}_{t,v}^{(m)}$ and $\hat{r}_{t,v}^{(m)}$, the optimal model of $\hat{F}_{t,v,y_p,y_a}^{(m)}$ for potters includes $\ln P_{t-3}$ and $\ln P_{t-4}$ as regressors but for divers, only $\ln P_{t-3}$. With exception to the constant term, further study of Table 6.1 shows that the estimated coefficients within fishing method are the same or very similar for the optimal models of both estimators. This
means that within fishing method, the optimal models between estimators, have similar trends but a different scale.

The optimal models of $\hat{\gamma}_o$ and $\hat{\gamma}_m$ are presented in Figures 6.1 – 6.4. Measured using MARB and MRB, the accuracy of these models in terms of model estimates and 1-step ahead forecasts is seen to be unaffected by the estimator used (Table 6.2).

Table 6.1: Estimated regression coefficients ($\beta_i$) for predictors ($x_i$) in the model $\hat{\gamma}_o = e^{\beta_0 + \beta_1 x_1 + \cdots + \beta_j x_j} + e_{i,t}$ using data for seasons 1986/87 – 2002/03, with $p_t$ and $r_t$ estimated using either $\hat{\gamma}_o$ or $\hat{\gamma}_m$. The standard error for each parameter is included in brackets. Different models of $\hat{\gamma}_o$ where various predictors are excluded, are considered and the optimal model (bold highlight) selected to minimize AICc. The significance level for the Ljung-Box statistic testing for evidence of any autocorrelation in the resulting error terms, for up to 5 time lags, is also presented.

<table>
<thead>
<tr>
<th>Fishing method</th>
<th>Estimator</th>
<th>Model</th>
<th>Predictors</th>
<th>AICc</th>
<th>L-Box ($P$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potting</td>
<td>$\hat{\gamma}_o$</td>
<td>I</td>
<td>Const.</td>
<td>ln $\hat{P}_{t-3}$</td>
<td>ln $\hat{P}_{t-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.24 (0.09)</td>
<td>0.16 (0.03)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>3.47 (0.13)</td>
<td>0.10 (0.04)</td>
<td>118.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III</td>
<td>3.14 (0.09)</td>
<td>0.15 (0.02)</td>
<td>0.06 (0.02)</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_m$</td>
<td>I</td>
<td>Const.</td>
<td>ln $\hat{P}_{t-3}$</td>
<td>ln $\hat{P}_{t-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.06 (0.09)</td>
<td>0.17 (0.03)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>3.29 (0.13)</td>
<td>0.11 (0.04)</td>
<td>113.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III</td>
<td>2.94 (0.08)</td>
<td>0.15 (0.02)</td>
<td>0.06 (0.02)</td>
</tr>
<tr>
<td>Diving</td>
<td>$\hat{\gamma}_o$</td>
<td>I</td>
<td>Const.</td>
<td>ln $\hat{P}_{t-3}$</td>
<td>ln $\hat{P}_{t-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.73 (0.16)</td>
<td>0.16 (0.04)</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>3.02 (0.16)</td>
<td>0.07 (0.05)</td>
<td>108.4</td>
</tr>
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<td></td>
<td></td>
<td>III</td>
<td>2.69 (0.16)</td>
<td>0.15 (0.04)</td>
<td>0.03 (0.24)</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_m$</td>
<td>I</td>
<td>Const.</td>
<td>ln $\hat{P}_{t-3}$</td>
<td>ln $\hat{P}_{t-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.56 (0.13)</td>
<td>0.16 (0.04)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>2.84 (0.15)</td>
<td>0.08 (0.05)</td>
<td>102.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III</td>
<td>2.50 (0.14)</td>
<td>0.15 (0.04)</td>
<td>0.04 (0.04)</td>
</tr>
</tbody>
</table>
Table 6.2: Comparisons of optimal models of $\hat{F}_{t,v; y_0, y_a}$ for each fishing method, potting and diving, where either $\hat{\gamma}_o$ or $\hat{\gamma}_m$ has been used to estimate $p_{t,v}^{(m)}$ and $r_{t,v}^{(m)}$. The accuracy of each model’s estimates and 1-step ahead forecasts is measured using MARB and MRB. Seasons 1987/88 – 2002/03 have been used to identify the optimal model and seasons 2003/04 – 2010/11, to compare model forecasts.

<table>
<thead>
<tr>
<th>Fishing method</th>
<th>Estimator</th>
<th>Estimates</th>
<th>Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MARB</td>
<td>MRB</td>
</tr>
<tr>
<td>Potting</td>
<td>$\hat{\gamma}_o$</td>
<td>7.9</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_m$</td>
<td>6.7</td>
<td>0.5</td>
</tr>
<tr>
<td>Diving</td>
<td>$\hat{\gamma}_o$</td>
<td>11.2</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_m$</td>
<td>10.0</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Figure 6.1: The model estimates and 1-step ahead forecasts of $\hat{F}_{t,v; y_0, y_a}$ for the optimal model $\hat{F}_{t,v; y_0, y_a}^{(m)} = e^{\eta_0 + \eta_1 \ln P_{t-3} + \eta_2 \ln P_{t-4} + \varepsilon_t}$ for potting. $\hat{F}_{t,v; y_0, y_a}^{(m)}$ has been calculated using $\hat{\gamma}_o$ to estimate both $p_{t,v}^{(m)}$ and $r_{t,v}^{(m)}$. The observed estimates of $\hat{F}_{t,v; y_0, y_a}^{(m)}$ for all seasons are also included as are the 95% confidence limits for the model estimates and forecasts, as estimated using Newton’s linearization method.
Figure 6.2: The model estimates and 1-step ahead forecasts of $\hat{F}_{t,v;g_{p,y_{0}}}$ for the optimal model $\hat{F}_{t,v;g_{p,y_{0}}} = e^{\alpha_{0} + \alpha_{1} \ln P_{t-3} + \alpha_{2} \ln P_{t-4}} + \varepsilon_{t,v}$ for potting. $\hat{F}_{t,v;g_{p,y_{0}}}$ has been calculated using $\hat{\gamma}_{m}$ to estimate both $\hat{p}_{t,v}^{(m)}$ and $\hat{r}_{t,v}^{(m)}$. The observed estimates of $\hat{F}_{t,v;g_{p,y_{0}}}$ for all seasons are also included as are the 95% confidence limits for the model estimates and forecasts, as estimated using Newton’s linearization method.
Figure 6.3: The model estimates and 1-step ahead forecasts of \( \hat{F}_{t,v;g,p,y_a}^{(m)} \) for the optimal model \( \hat{F}_{t,v;g,p,y_a}^{(m)} = e^{a_0 + a_1 \ln P_{t-3} + a_2 \ln P_{t-4} + \epsilon_{t,v}} \) for diving. \( \hat{F}_{t,v;g,p,y_a}^{(m)} \) has been calculated using \( \hat{\gamma}_0 \) to estimate both \( p_{t,v}^{(m)} \) and \( r_{t,v}^{(m)} \). The observed estimates of \( \hat{F}_{t,v;g,p,y_a}^{(m)} \) for all seasons are also included as are the 95% confidence limits for the model estimates and forecasts, as estimated using Newton’s linearization method.
Figure 6.4: The model estimates and 1-step ahead forecasts of $\hat{F}_{t,v,y_0,y_0}^{(m)}$ for the optimal model $\hat{F}_{t,v,y_0,y_0}^{(m)} = \gamma_0 + \alpha_1 \ln P_{t-1} + \alpha_2 \ln P_{t-4} + \varepsilon_{t,v}$ for diving. $\hat{F}_{t,v,y_0,y_0}^{(m)}$ has been calculated using $\gamma_m$ to estimate both $\hat{p}_{t,v}^{(m)}$ and $\hat{r}_{t,v}^{(m)}$. The observed estimates of $\hat{F}_{t,v,y_0,y_0}^{(m)}$ for all seasons are also included as are the 95% confidence limits for the model estimates and forecasts, as estimated using Newton’s linearization method.
Modelling licence usage $\hat{L}_{t,v,y_p}^{(m)}$

The MS estimates $\hat{L}_{t,v,y_p}^{(m)}$ calculated using either $\hat{h}_o$ or $\hat{h}_m$ to estimate $\hat{p}_{t,v}^{(m)}$, are presented in Figure 6.5. The total number of people licenced to fish in each season (umbrella + RL specific licence holders), as well as the number of those that only held a RL specific licence, is also illustrated.

A study of the ACF and PACF plots of $\hat{L}_{t,v,y_p}^{(m)}$ for potters and divers calculated using $\hat{h}_o$ (Figures 6.6 – 6.9) suggest an AR(1) process for both fishing methods. The ACF and PACF plots of $\hat{L}_{t,v,y_p}^{(m)}$ for potters and divers calculated using $\hat{h}_m$ also suggest an AR(1) process (Appendix F). It was decided to consider adjacent ARMA models up to the AR(2) for $\hat{L}_{t,v,y_p}^{(m)}$, for both fishing methods and estimators (Table 6.3). Under the AICc criterion, it is determined that the AR(1) process is optimal for modelling $\hat{L}_{t,v,y_p}^{(m)}$ calculated using either $\hat{h}_o$ or $\hat{h}_m$ for both potting and diving.

Assuming an AR(1) process, the usefulness of including various puerulus indices as regressors is investigated (Table 6.4). For potters, including $\ln P_{t-1}$ as a regressor was optimal for the number of potters estimated using $\hat{h}_o$ (AICc = -8.96, Table 6.4) and $\hat{h}_m$ (AICc = -9.28, Table 6.4). For divers, including none of the considered regressors was optimal (Table 6.4: $\hat{h}_o$, AICc = -4.18; $\hat{h}_m$, AICc = -4.28). The optimal models of $\hat{L}_{t,v,y_p}^{(m)}$ for both fishing methods, estimated using either of the two estimators, are presented in Figures 6.10 – 6.13. For completeness, the parameters for each of these models are presented in Table 6.5 and show that within fishing method, the estimated parameters between estimators ($\hat{h}_o$ and $\hat{h}_m$) are similar. These models perform similarly, in terms of MARB and MRB, under each estimator for a given fishing method (Table 6.6) in terms of both model estimates and 1-step ahead forecasts.

Note that the AICc values for corresponding models in Table 6.3 and Table 6.4 differ. Table 6.3 compared ARMA models of different orders and required restricting the modelled series to be the same for all models so AICc could be used to choose between models. In Table 6.4 only AR(1) models were compared and hence, all of
the data for 1986/87 – 2002/03 was used. The difference in AICc values is thus, the result of increasing the number of modelled data points.

Figure 6.5: The estimated number of licence holders that fished for rock lobster each season by (A) potting and (B) diving. Using the mail survey data, these estimates are the result of applying either the ratio ($\hat{\gamma}_p$) or multinomial ($\hat{\gamma}_m$) to estimate the proportion of total licensees that used each fishing method. The total number of people licensed to fish for rock lobster (umbrella and rock lobster specific licence holders) each season, as well as those that only held a rock lobster specific (RL specific) licence, are also presented. It should be noted that some licence holders caught lobsters by both potting and diving and hence, would be recorded as fishers in both panels A and B.
Figure 6.6: Autocorrelations (ACF) of $\ln \hat{I}_{t,v,\theta_p}$ for each lag where $\hat{I}_{t,v,\theta_p}$ is the number of potters estimated by applying $\hat{\gamma}_o$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).

Figure 6.7: Partial autocorrelations (PACF) of $\ln \hat{I}_{t,v,\theta_p}$ for each lag where $\hat{I}_{t,v,\theta_p}$ is the number of potters estimated by applying $\hat{\gamma}_o$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).
Figure 6.8: Autocorrelations (ACF) of $\ln \hat{L}_{t,v,y_p}^{(m)}$ for each lag where $\hat{L}_{t,v,y_p}^{(m)}$ is the number of divers estimated by applying $\hat{\gamma}_\alpha$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).

Figure 6.9: Partial autocorrelations (PACF) of $\ln \hat{L}_{t,v,y_p}^{(m)}$ for each lag where $\hat{L}_{t,v,y_p}^{(m)}$ is the number of divers estimated by applying $\hat{\gamma}_\alpha$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).
Table 6.3: Various description statistics resulting from fitting different ordered models of ARMA(q1,q2) to \( \ln \hat{L}_{t,v,y}^{(m)} \), \( v \in \{ \text{pot}, \text{dive} \} \), using the data of seasons up to and including 2002/03. \( \hat{L}_{t,v,y}^{(m)} \) is calculated by using \( \hat{\gamma}_o \) or \( \hat{\gamma}_m \) to estimate \( p_{t,v}^{(m)} \). The optimal model under each estimator and fishing method, based on minimum AICc, are highlighted in bold. Non-invertible models are highlighted by *.

<table>
<thead>
<tr>
<th>Fishing method</th>
<th>Estimator</th>
<th>Seasons Used</th>
<th>ARMA ((q_1,q_2))</th>
<th>AICc</th>
<th>AR roots</th>
<th>MA roots</th>
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<tbody>
<tr>
<td>Potting</td>
<td>( \hat{\gamma}_o )</td>
<td>88/89 – 02/03</td>
<td>(0,0)</td>
<td>3.55</td>
<td>-</td>
<td>-</td>
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<td></td>
<td></td>
<td>87/88 – 02/03</td>
<td>(1,0)</td>
<td>-8.87</td>
<td>1.42</td>
<td>-</td>
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<td></td>
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<td>87/88 – 02/03</td>
<td>(0,1)</td>
<td>-5.53</td>
<td>-</td>
<td>1.00*</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>-6.89</td>
<td>1.15</td>
<td>4.23</td>
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<tr>
<td></td>
<td></td>
<td>86/87 – 02/03</td>
<td>(2,0)</td>
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<td>1.26, 5.79</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( \hat{\gamma}_m )</td>
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<td>-</td>
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<td>-</td>
<td>1.00*</td>
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<td>-5.90</td>
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<td></td>
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<td>-4.85</td>
<td>1.36, 47.70</td>
<td>-</td>
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<tr>
<td>Diving</td>
<td>( \hat{\gamma}_o )</td>
<td>88/89 – 02/03</td>
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<td>18.31</td>
<td>-</td>
<td>-</td>
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<td></td>
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<td>(1,0)</td>
<td>-9.08</td>
<td>1.16</td>
<td>-</td>
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<td>(0,1)</td>
<td>8.64</td>
<td>-</td>
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<td>1.09</td>
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<td>-6.22</td>
<td>1.33, 2.70</td>
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Table 6.4: The AICc for various AR(1) models of $\ln \hat{L}^{(m)}_{t,v,y}$, $v \in \{pot, dive\}$, using the data of seasons 1986/87 – 2002/03, with and without puerulus settlement indices $\ln P_{t-3}$ and $\ln P_{t-4}$ included as regression variables. $\hat{\gamma}_{o}$ and $\hat{\gamma}_{m}$ are calculated by using $\hat{\gamma}_{o}$ or $\hat{\gamma}_{m}$ to estimate $p_{t,v}^{(m)}$. The optimal model under each estimator and fishing method, based on minimum AICc, are highlighted in bold. The root of the characteristic polynomial of the AR component of the model is also presented to assess stationarity (all models are stationary).

<table>
<thead>
<tr>
<th>Fishing method</th>
<th>Estimator</th>
<th>Regressors</th>
<th>AICc</th>
<th>Roots</th>
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<tr>
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<td>-</td>
<td>-8.85</td>
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<td></td>
<td>$\ln P_{t-3}$</td>
<td>-7.01</td>
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<td>$\ln P_{t-4}$</td>
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<td><strong>1.19</strong></td>
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<td></td>
<td></td>
<td>$\ln P_{t-3} \cdot \ln P_{t-4}$</td>
<td>-6.56</td>
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<td>$\hat{\gamma}_{m}$</td>
<td>-</td>
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<td>1.27</td>
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<td></td>
<td>$\ln P_{t-3}$</td>
<td>-7.31</td>
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<td>$\ln P_{t-4}$</td>
<td><strong>-9.28</strong></td>
<td><strong>1.20</strong></td>
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<td></td>
<td></td>
<td>$\ln P_{t-3} \cdot \ln P_{t-4}$</td>
<td>-7.62</td>
<td>1.26</td>
</tr>
<tr>
<td>Diving</td>
<td>$\hat{\gamma}_{o}$</td>
<td>-</td>
<td><strong>-4.18</strong></td>
<td><strong>1.06</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\ln P_{t-3}$</td>
<td>-2.33</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\ln P_{t-4}$</td>
<td>-2.09</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\ln P_{t-3} \cdot \ln P_{t-4}$</td>
<td>0.06</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_{m}$</td>
<td>-</td>
<td><strong>-4.28</strong></td>
<td><strong>1.07</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\ln P_{t-3}$</td>
<td>-3.10</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\ln P_{t-4}$</td>
<td>-1.99</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\ln P_{t-3} \cdot \ln P_{t-4}$</td>
<td>-0.51</td>
<td>1.09</td>
</tr>
</tbody>
</table>
Table 6.5: Parameter estimates for the optimal models identified for modelling $\ln \widehat{L}_{t,v;\hat{y}_{p}}^{(m)}$. Parameters have been estimated using seasons 1986/87 – 2002/03 only. Standard errors for the estimates are included in brackets.

<table>
<thead>
<tr>
<th>Fishing method</th>
<th>Estimator</th>
<th>$\phi_1$</th>
<th>$\ln P_{1-t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potting</td>
<td>$\widehat{\gamma}_o$</td>
<td>0.8418 (0.1626)</td>
<td>0.0706 (0.0402)</td>
</tr>
<tr>
<td></td>
<td>$\widehat{\gamma}_m$</td>
<td>0.8330 (0.1383)</td>
<td>0.0787 (0.0400)</td>
</tr>
<tr>
<td>Diving</td>
<td>$\widehat{\gamma}_o$</td>
<td>0.9476 (0.0799)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\widehat{\gamma}_m$</td>
<td>0.9368 (0.0875)</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.6: Comparisons of optimal models for $\widehat{L}_{t,v;\hat{y}_{p}}^{(m)}$, $v \in \{\text{pot, dive}\}$, where $p_{t,v}^{(m)}$ is estimated by either $\widehat{\gamma}_o$ or $\widehat{\gamma}_m$. The accuracy of each model's estimates and 1-step ahead forecasts is measured using MARB and MRB. Seasons 1987/88 – 2002/03 have been used to compare model estimates and seasons 2003/04 – 2010/11, to compare the model forecasts.

<table>
<thead>
<tr>
<th>Fishing method</th>
<th>Estimator</th>
<th>Estimates</th>
<th>Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MARB</td>
<td>MRB</td>
</tr>
<tr>
<td>Potting</td>
<td>$\widehat{\gamma}_o$</td>
<td>12.3</td>
<td>-2.5</td>
</tr>
<tr>
<td></td>
<td>$\widehat{\gamma}_m$</td>
<td>12.0</td>
<td>-2.1</td>
</tr>
<tr>
<td>Diving</td>
<td>$\widehat{\gamma}_o$</td>
<td>15.8</td>
<td>-4.3</td>
</tr>
<tr>
<td></td>
<td>$\widehat{\gamma}_m$</td>
<td>15.1</td>
<td>-4.1</td>
</tr>
</tbody>
</table>
Figure 6.10: The model estimates and 1-step ahead forecasts of $\hat{L}_{t, v; y_p}^{(m)}$, $v = \text{potting}$ and $y_p = \hat{f}_o$, using the optimal AR(1) model with regressor $\ln P_{t-4}$. The 95% confidence limits for these estimates and forecasts, as estimated using Newton’s linearization method, are included along with all observed estimates of $\hat{L}_{t, v; y_p}^{(m)}$.

Figure 6.11: The model estimates and 1-step ahead forecasts of $\hat{L}_{t, v; y_p}^{(m)}$, $v = \text{potting}$ and $y_p = \hat{f}_m$, using the optimal AR(1) model with regressor $\ln P_{t-4}$. The 95% confidence limits for these estimates and forecasts, as estimated using Newton’s linearization method, are included along with all observed estimates of $\hat{L}_{t, v; y_p}^{(m)}$. 
Figure 6.12: The model estimates and 1-step ahead forecasts of $\hat{L}_{t,v,y_0}^{(m)}$, $v = \text{diving}$ and $y_0 = \hat{\gamma}_v$, using the optimal AR(1) model with no regressor. The 95% confidence limits for these estimates and forecasts, as estimated using Newton’s linearization method, are included along with all observed estimates of $\hat{L}_{t,v,y_0}^{(m)}$.

Figure 6.13: The model estimates and 1-step ahead forecasts of $\hat{L}_{t,v,y_0}^{(m)}$, $v = \text{diving}$ and $y_0 = \hat{\gamma}_v$, using the optimal AR(1) model with no regressor. The 95% confidence limits for these estimates and forecasts, as estimated using Newton’s linearization method, are included along with all observed estimates of $\hat{L}_{t,v,y_0}^{(m)}$. 

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Assessing $\widehat{T}_t^{(m)}$ and $\widehat{T}_t^{(m)}$ to model seasonal total catch $\widehat{\gamma}_t^{(m)}$

The degree to which the identified optimal models describing $\widehat{T}_t^{(m)}$ and $\widehat{T}_t^{(m)}$ for potting (Figures 6.14 & 6.15) and diving (Figures 6.16 & 6.17), estimate and forecast the observed data are similar between estimators for each fishing method (Table 6.7). It is also seen that that the 95% confidence intervals of the PDS total catch estimates have greater overlap with those resulting from the MS method using $\widehat{\gamma}_m$ (Figures 6.15 & 6.17) than when using $\widehat{\gamma}_o$ (Figures 6.14 & 6.16), although there clearly exists a consistent bias between the two estimates i.e. MS estimate are always higher than the PDS.

**Figure 6.14**: The model estimates and 1-step ahead forecasts of $\widehat{T}_t^{(m)}$, using the optimal models identified for $\widehat{T}_t^{(m)}$ and $\widehat{T}_t^{(m)}$, along with 95% confidence limits estimated using Newton’s linearization method derived. The observed estimates of $T_t^{(m)}$ are also included as are the phone-diary survey estimates and their 95% percentile bootstrap confidence limits.
Figure 6.15: The model estimates and 1-step ahead forecasts of $\hat{T}_{t,\text{pot},m,m}^{(m)}$, using the optimal models identified for $\hat{T}_{t,\text{pot},m}$ and $\hat{F}_{t,\text{pot},m,m}$, along with 95% confidence limits estimated using Newton’s linearization method derived. The observed estimates of $\hat{T}_{t,\text{pot},m,m}$ are also included as are the phone-diary survey estimates and their 95% percentile bootstrap confidence limits.
Figure 6.16: The model estimates and 1-step ahead forecasts of $\hat{T}_{t,\text{dive, } o, o'}^{(m)}$ using the optimal models identified for $\hat{T}_{t,\text{dive, } o}$ and $\hat{T}_{t,\text{dive, } o, o'}^{(m)}$, along with 95% confidence limits estimated using Newton’s linearization method derived. The observed estimates of $\hat{T}_{t,\text{dive, } o, o'}^{(m)}$ are also included as are the phone-diary survey estimates and their 95% percentile bootstrap confidence limits.
Figure 6.17: The model estimates and 1-step ahead forecasts of $\hat{T}_{t,\text{dive},m,n}$, using the optimal models identified for $\hat{T}_{t,\text{dive},m}$ and $\hat{T}_{t,\text{dive},m,n}$, along with 95% confidence limits estimated using Newton’s linearization method derived. The observed estimates of $\hat{T}_{t,\text{dive},m,n}$ are also included as are the phone-diary survey estimates and their 95% percentile bootstrap confidence limits.

Table 6.7: Comparisons of optimal models for $\hat{T}_{t,v,\gamma_p,y_0}$, where $p^{(m)}_{t,v}$ and $r^{(m)}_{t,v}$ are both estimated by either $\hat{\gamma}_o$ or $\hat{\gamma}_m$. The accuracy of each model’s estimates and 1-step ahead forecasts is measured using MARB and MRB. Seasons 1987/88 – 2002/03 have been used to compare model estimates and seasons 2003/04 – 2010/11, to compare the model forecasts.

<table>
<thead>
<tr>
<th>Fishing method</th>
<th>Estimator</th>
<th>Estimates</th>
<th>Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MARB</td>
<td>MRB</td>
</tr>
<tr>
<td>Potting</td>
<td>$\hat{\gamma}_o$</td>
<td>13.4</td>
<td>-1.0</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_m$</td>
<td>12.7</td>
<td>-0.8</td>
</tr>
<tr>
<td>Diving</td>
<td>$\hat{\gamma}_o$</td>
<td>19.2</td>
<td>-1.8</td>
</tr>
<tr>
<td></td>
<td>$\hat{\gamma}_m$</td>
<td>17.7</td>
<td>-2.2</td>
</tr>
</tbody>
</table>
6.3 Discussion

The optimal regression modelling \( \hat{F}_{t,pot,3y,3y}^{(m)} \) when using either \( \hat{\gamma}_o \) or \( \hat{\gamma}_m \) to estimate \( p_{t,v} \) and \( r_{t,v} \) only differed by a scaling factor. The same was true for the models of \( \hat{F}_{t,div,3y,3y}^{(m)} \). Comparing the optimal models between fishing method, it is seen that \( \hat{F}_{t,pot,3y,3y}^{(m)} \) was dependent on lobster settlement 3 and 4 years prior whilst \( \hat{F}_{t,div,3y,3y}^{(m)} \) was dependent only on that 3 years prior. This is biologically reasonable given legally sized lobsters in the shallow near shore waters where divers would more safely fish, are more likely to be faster growing 3 year olds. Potters can more safely fish to deeper depths further off the coast and thus, also have greater access to the slower growing lobsters that generally do not enter the fishery until they have began their migration to further off the coast after the ‘whites’ moult that occurs 4 years post settlement (see Section 2.2 in Chapter 2 for further details).

Within fishing method, the optimal time series models describing \( \hat{L}_{t,pot,3y}^{(m)} \) were similar when using either \( \hat{\gamma}_o \) or \( \hat{\gamma}_m \) to estimate \( p_{t,v} \), with an AR(1) best describing \( \hat{L}_{t,pot,3y}^{(m)} \) for both potting and diving. Including regressors into the AR(1) model, it was seen that \( \hat{L}_{t,pot,3y}^{(m)} \) was dependent on puerulus settlement 4 years prior but \( \hat{L}_{t,div,3y}^{(m)} \) was not influenced by puerulus settlement at all. The catch rates each season are at their highest in the early parts of the season (November/December) when the ‘whites’ are migrating and are more catchable. Being influenced by the puerulus settlement 4 years prior therefore suggests that people are more likely to go potting when lobsters are more abundant and much easier to catch. This information is readily available to fishers prior and during the course of the season through reports in angler magazines and local newspapers. Presumably due to the leisure activity of diving in itself, the nonsignificance of lobster abundance in modeling \( \hat{L}_{t,div}^{(m)} \) suggests that people are as likely to go diving for lobsters in poor recruitment years as they are in ‘good’.

Umbrella licences were introduced at the start of the 1992/93 season and a sharp rise in the number of licensees able to fish for rock lobster was expected due to people with interests in other fisheries purchasing this licence for possible cost savings. A significant rise in the number of licensees did not occur however, until the 1997/98
season and may have been delayed due to a lack of awareness about the availability and economic benefits of the umbrella licence at the early stages of its introduction. The confidence intervals around the model estimates of \( \hat{\gamma} \) for each fishing method appear to be very conservative and whether this is due to this regime shift not being accounted for in the model, or the method used to approximation them, is unclear and requires further investigation. The point estimates and forecasts however, were reasonable.

In terms of interpretation and accuracy, the optimal models for \( \hat{\gamma}_{t,v,y,p} \) and \( \hat{\gamma}_{t,v,d,y,a} \) were unaffected by the choice of \( \hat{\gamma}_o \) or \( \hat{\gamma}_m \) to estimate \( p_{t,v} \) and \( r_{t,v} \). Whilst applying \( \hat{\gamma}_m \) rather than \( \hat{\gamma}_o \) to the MS data leads to total catch estimates that are much more similar to those of the PDS, there remains a positive bias that still needs to be corrected. If future work to adjust for this likely recall bias leads to even further agreement between the MS and PDS total catch estimates by using \( \hat{\gamma}_m \), then our understanding of the drivers influencing the trend of this fishery are unlikely to change.

The western rock lobster fishery is currently managed under an integrated fisheries management (IFM) framework where the recreational catch take is limited to 5% of the total catch (recreational + commercial). The identification in this research that people are less likely to pot for lobsters when they are less abundant, a fishing method that accounts for a significant proportion of the recreational catch, means that the recreational take as a proportion of the total is minimized (maximized) in lower (higher) abundance years. This suggests that if managers want to maintain a fixed proportion of share allocation between the two sectors on a season by season basis, then employing a conservative management policy in higher catch years (e.g. lower bag limits and restricted effort), but with a slackening of these restrictions in leaner catch years, may be necessary.
Chapter 7

General Discussion

Early in 2000 the western rock lobster fishery was earmarked to operate under an IFM framework that would allocate catch share between the commercial and recreational sectors. To determine this allocation the historical catch share between these two sectors was to be used as a guide. The historical catches for the commercial sector were known due to mandatory reporting systems for the entire fleet. The recreational catches however were considered less reliable given they had been estimated using data collected from an end of season mail survey (MS) that had a nonresponse rate of between 40% and 60% for any one season.

Given the concern surrounding the nonresponse rates of the annual end of season MS of people licensed to fish recreationally for rock lobster, a generally considered less biased, but much more expensive, phone-diary survey (PDS) was also run for the 2000/01 and 2001/02 seasons. By running both the MS and PDS in the same seasons, managers were hoping that a correction factor could be identified that reliably removed any bias in the time series of estimates from the MS that were available since the 1986/87 season. PDSs were again run in seasons 2004/05 through to 2008/09 to extend the available data for estimating this correction factor.

Linear regressions describing the PDS estimates in terms of those from the MS were fitted for both average retained catch per fisher ($\tilde{F}_{t,v}$) and the estimated number of people who fished ($\tilde{L}_{t,v}$). These regressions identified that the MS and PDS estimates for both parameters differed by only a scaling factor. Multiplying these two scaling factors together, a single correction factor was constructed to standardize the total catch estimates resulting from the MS to the scale of the generally accepted, more accurate PDS.

In the process of identifying a correction factor, and under the assumption that the PDS is more accurate than the MS, it was observed that non-fishers are less likely to
return the MS form and that for fishers, the probability of returning the survey form increases with their reported number of days fished. These tendencies of highly ‘interested’ or ‘involved’ people in the surveyed topic being more likely to return a survey than someone less so, have also been observed in other studies (e.g. van Kenhove et al. 2002; Brown & Wilkins 1978), and largely explains why the unadjusted MS estimates of total catch are much higher than for the corresponding PDS.

The unadjusted total catch estimates of the MS are estimated using effort (the proportion of people who fished) calculated by the ratio estimator ($\hat{\gamma}_0$) that estimates the proportion of the population who fished as the proportion of respondents who fished. With fishers being more likely to return a MS form than non-fishers, this means using $\hat{\gamma}_0$ leads to overestimates in effort. It was then decided to investigate competing estimators to $\hat{\gamma}_0$ that were less affected by this type of bias. By using these estimators in place of $\hat{\gamma}_0$ it was hoped that the required frequency of running the more accurate but usually more expensive surveys to estimate the correction factor, could be reduced. These estimators could also be generally applied to MSs of other research fields unlike the correction factor estimated in this thesis which is only applicable to the recreational western rock lobster MS and even then, is known to be ‘good’ only for the seasons from which it was calculated.

In estimating the proportion of the population that has some characteristic ($\gamma$), $\hat{\gamma}_o$ assumes that the probability of those with the characteristic $x$ returning the survey form ($\pi_x$) is the same as those without ($\pi_{\bar{x}}$) i.e. $\pi_x = \pi_{\bar{x}}$. The expectation ($\hat{\gamma}_e$) and multinomial ($\hat{\gamma}_m$) estimators are constructed and differ in their assumed values of $\pi_x$ and $\pi_{\bar{x}}$. Under the assumption that $\pi_x > \pi_{\bar{x}}$, $\gamma_m$ chooses $\pi_x$ and $\pi_{\bar{x}}$ to minimize it’s expected root-mean-square-error (RMSE) in estimating $\gamma$, and $\hat{\gamma}_e$ sets $\pi_x$ and $\pi_{\bar{x}}$ at their expected value given the number of returned surveys.

The construction of $\hat{\gamma}_m$ and $\hat{\gamma}_e$ required distributions for $\pi_x$ and $\pi_{\bar{x}}$. This research assumed these to be uniformly distributed with Manski’s Bounds (Manski 1995) as boundary points. Natural boundary points of 0 and 1 could have been used but using Manski’s Bounds takes advantage of a survey’s higher return rate to reduce the range
of possible values for $\pi_x$ and $\pi_\mp$ given the observed survey responses. Assuming that the possible values of $\pi_x$ and $\pi_\mp$ are equally likely, the uniform distribution was considered appropriate for the purpose of generalizing the newly identified estimators for use in MSs of other fisheries or fields of study. The $\hat{\gamma}_m$ and $\hat{\gamma}_e$ could be derived using other distributions for the response probabilities, including any of those used in the various model-based approaches in the literature (e.g. Stasny 1991; Nandram & Choi 2002), if this was considered beneficial for the survey returns of a particular study.

Assuming the response probabilities are uniformly distributed with Manski’s Bounds determining the boundary points, $\hat{\gamma}_m$ and $\hat{\gamma}_e$ were constructed along with their approximate variance functions and the resulting Clopper-Pearson confidence intervals were shown to have approximately the nominated level of coverage. In terms of RMSE, simulations were used to determine when $\hat{\gamma}_m$ and $\hat{\gamma}_e$ were better than $\hat{\gamma}_o$. The $\hat{\gamma}_e$ outperformed $\hat{\gamma}_o$ when $\pi_x$ was at least marginally greater than $\pi_\mp$. The required difference between $\pi_x$ and $\pi_\mp$ for $\hat{\gamma}_m$ to be the best performing of the three estimators increased with $\gamma$. This required difference between $\pi_x$ and $\pi_\mp$ had the benefit however, of $\hat{\gamma}_m$ producing even greater reductions in RMSE than $\hat{\gamma}_e$.

In practice it would be difficult to determine the use of $\hat{\gamma}_m$ over $\hat{\gamma}_e$ given the dependence on the difference between the response probabilities for a given level of $\gamma$. Instead, it is much simpler to base that decision on the range of $\gamma$ alone. Averaging the achieved reduction in bias over all $\pi_x$ and $\pi_\mp$ (such that $\pi_x > \pi_\mp$), $\hat{\gamma}_m$ outperforms $\hat{\gamma}_o$ and $\hat{\gamma}_e$ when $\gamma \leq 0.65$. Over this range it is seen that $\hat{\gamma}_o$ produces an average relative bias of between 25% and 131% and $\hat{\gamma}_m$ reduces these by between 53% and 100%. Over the same range, using $\hat{\gamma}_e$ leads to reductions of between 48% and 61%. Although the percentage reduction in bias of $\hat{\gamma}_e$ is lower than that achieved by $\hat{\gamma}_m$ when $\gamma \leq 0.65$, it does have the advantage of not running the risk of increasing bias. For $\gamma > 0.65$, $\hat{\gamma}_m$ increases the average relative bias in $\hat{\gamma}_o$ by as much as 333% whilst $\hat{\gamma}_e$ continues to reduce, although the achieved reductions in actual bias in this range are small.
In the real world for a real MS, $\gamma$ is unknown and so methods are required to reliably determine if $\gamma_m$ is appropriate (i.e. $\gamma \leq 0.65$) to achieve maximum reductions in nonresponse bias but at the same time, avoiding the risk of actually adding further bias to resulting estimates (when $\gamma > 0.65$). For a MS of a fishery that does not require people to be licensed, the telephone book (‘white pages’) might be used as a reference frame. Given that this reference frame is not as targeted as say, using a licence database, it might be reasonably assumed that the probability of someone in the surveyed population having fished is ‘low’ and hence, $\gamma_m$ is appropriate for estimating this parameter. A mathematical approach for determining that $\gamma \leq 0.65$ involves using $\gamma_o$ as a guide. Under the assumption that $\pi_v > \pi_T$ then $\gamma_o$ is an upper limit of $\gamma$. Therefore, if $\gamma_o \leq 0.65$ then $\gamma \leq 0.65$ necessarily follows. If however, $\gamma_o > 0.65$ then this method is inconclusive.

The time series of western rock lobster MS data was used to demonstrate how to choose between $\gamma_v$ and $\gamma_m$ in a real problem. To maximize the opportunity of reducing the nonresponse bias in the total catch estimates, the total catch equation was formulated to include not only fishing participation ($p_{t,v}$) but also the proportion of these fishers that were ‘avid’ ($r_{t,v}$). For both fishing methods, the estimate of $p_{t,v}$ using $\gamma_o$ indicated that $p_{t,v} \leq 0.65$ and hence, $\gamma_m$ was used to estimate $p_{t,v}$ and resulted in estimated number of fishers ($\hat{L}_{t,v}$) for each fishing method being very similar to the corresponding estimates from the PDS.

The value of $r_{t,v}$ is dependent on the definition of avidity (avid fished $> a_v$ days). To define $a_v$ for the western rock lobster fishery, the proportion of fishers that reported fishing more than a set number of days were compared between the PDS and MS. So that maximum reductions of avidity bias in $\hat{F}^{(m)}_{t,v}$ could be achieved, these comparisons were used to define $a_v$, so that $\gamma_m$ was most appropriate for estimating $r_{t,v}$ from the MS. A sensitivity analysis for the rock lobster MS demonstrated that the definition of avidity did not have a great bearing on resulting total catch estimates. In fact the choice of adjusting for an avidity bias, and the estimator ($\gamma_v$ or $\gamma_m$) chosen to do this, had the greatest impacts on these estimates with the definition itself leading to only minor adjustments thereafter.
Defining avidity subject to the criterion $\pi_a > \pi_S$ only, and then using $\hat{\gamma}_o$ to test the appropriateness of $\hat{\gamma}_c$ or $\hat{\gamma}_m$ to estimate $r_{t,v}$ was also possible. By adding the restriction that $r_{t,v} \leq 0.65$ however, ensured that the reduction in nonresponse bias for estimates of $F_{t,v}$ from the MS was maximized.

A significant improvement in $\hat{F}_{t,v}$ (a function of $\hat{r}_{t,v}^{(m)}$) was achieved by estimating $r_{t,v}^{(m)}$ using $\hat{\gamma}_m$, although the reduction in bias, measured by comparing to estimates from the PDS, was not as successful as for $\hat{L}_{t,v}$. This difference in success may be due to a recall bias where the inaccurate reporting of days fished may result in fishers being misallocated to an avidity class where as a person is more likely to accurately recall whether or not they did go fishing. Future research should attempt to describe the likely recall bias in a persons number of days fished.

In a comparison of a MS and PDS of anglers, Connelly & Brown (1995) provides evidence that fishers are able to accurately recall a low number of days fished but tend to overestimate this effort as the true level grows. Citing other research, Connelly & Brown (1995) explains this recall accuracy in terms of how people might make these estimates; people may attempt to recall individual fishing events when they fish infrequently (episode enumeration) but may use rule-based enumeration, such as multiplying their average recall of days fished per month by the number of months in a season, as their number of days fished in a season increases.

If fishers use rule-based enumeration to estimate their number of fishing days then they might also use a similar rule for estimating their catch e.g. multiply their estimated number of fishing days by their recall of on average, how many lobsters they retained per trip. The reported daily catch rates in the MS of recreational western rock lobster fishers were highly correlated with indices of lobster abundance. It was also seen that for a given level of days fished, the MS and PDS produced similar estimates of average retained catch per fisher between seasons. Therefore, if fishers are using rule-based enumeration to estimate their catches, there is good evidence to support that fishers are able to accurately recall their average catch per trip in a MS. By identifying a relationship between recall bias and a fishers estimate of days fished, not only should their reported number of days fished be adjusted but
also should their total catch estimate commensurate with the percentage change in their level of effort. These adjustments would not only reduce the impact of recall bias on the average catch per fisher but would also adjust the observed avidity rate. Making these corrections and using estimators such as $\hat{\gamma}_c$ and $\hat{\gamma}_m$ may lead to estimates of $\hat{F}_{t,v}^{(m)}$ that are more similar to those from the PDS.

With exception to a scaling factor for $\hat{F}_{t,v}^{(m)}$, optimum models within fishing method describing $\hat{L}_{t,v}^{(m)}$ and $\hat{F}_{t,v}^{(m)}$, calculated using $\hat{\gamma}_o$ or $\hat{\gamma}_m$ to estimate $p_{t,v}^{(m)}$ and $r_{t,v}^{(m)}$, were similar. For both fishing methods, $\hat{F}_{t,v}^{(m)}$ is well explained by an exponential and $\hat{L}_{t,v}^{(m)}$ by an autoregressive model of order 1. Including a measure of lobster abundance improved the models of $\hat{F}_{t,v}^{(m)}$ but interestingly, lobster abundance only improved the model of $\hat{L}_{t,v}^{(m)}$ for potters and not divers. This suggests that people are more likely to put pots in the water when lobsters are more abundant and easier to catch but, presumably due to the leisure activity of diving in itself, are still as likely to go diving even in years when lobsters are less abundant. The retained catches of potters were driven by lobster abundance in both deep and shallow water (3 and 4 year old post settlement lobsters) whilst those of divers were driven by animals more likely in the shallow waters (3 year old lobsters). The conclusions are biologically reasonable given potters have better access to deeper waters whilst divers are more likely to be constrained to their depth of fishing.

Differing interpretations of models describing $\hat{L}_{t,v}^{(m)}$ and $\hat{F}_{t,v}^{(m)}$ using $\hat{\gamma}_o$ or $\hat{\gamma}_m$ would have been a concern. If future work to adjust for the recall bias whilst using $\hat{\gamma}_m$ to estimate $p_{t,v}$ and $r_{t,v}$ leads to similar catch estimates to those using $\hat{\gamma}_o$ and then multiplying by a correction factor (as done in Chapter 3), then the interpretation of the models for $\hat{L}_{t,v}^{(m)}$ and $\hat{F}_{t,v}^{(m)}$ (under $\hat{\gamma}_m$) will remain the same and only the scale will change.

For $\hat{\gamma}_m$ and $\hat{\gamma}_e$ to better estimate $\gamma$ than $\hat{\gamma}_o$, $\pi_c > \pi_f$ must hold. In this research the PDS was compared to the MS to determine if this condition was true. Access to a secondary, more reliable data set may not exist for the MS of another fishery. In such
cases, determining if $\pi_f > \pi_T$ or what level for $\alpha_e$ leads to $\pi_a > \pi_R$, requires an alternative method such as the resampling method proposed by Srinath (1971).

After an initial mail-out of a survey form Srinath (1971) proposed further waves of sampling by resending the form to a randomly selected smaller sample of nonrespondents to the previous until in the last wave, no nonrespondents occur. The responses to each wave of sampling are then combined to give estimates that account for the nonresponse bias inherent in the initial wave. Total catch estimates resulting from such a sampling method may have an undesirable low level of precision. This method may however produce adjusted estimates of $\hat{p}_{t,o}^{(m)}$ and $\hat{r}_{t,o}^{(m)}$ that are useful for determining if $\pi_e > \pi_T$ is likely true in the first wave and hence, whether or not $\hat{\gamma}_m$ and $\hat{\gamma}_e$ should be applied.

The proposed method of sampling by Srinath (1971) minimizes survey costs by surveying only a proportion of the respondents at each successive wave of sampling. In an attempt to increase response rates to the MS after the first wave, a shorter survey form designed to only provide information for assessing if $\pi_e > \pi_T$ could be used. Sending a shorter survey form requires less respondent burden and should result in higher return rates (Dillman 1991) to minimize the required number of waves and hence, further reduce costs.

In this research $\hat{\gamma}_e$ and $\hat{\gamma}_m$ have been constructed under the assumption that $\pi_e > \pi_T$. If for a particular study this assumption is not true, then versions of $\hat{\gamma}_e$ and $\hat{\gamma}_m$ could be derived under this opposing assumption i.e. $\pi_e < \pi_T$. Thus, assuming $\pi_e > \pi_T$ should not be seen as a limitation to this research.

MSs have been used to estimate the recreational catch and effort by many fisheries around the world (e.g. Tarrant & Manfredo 1993; Guillory 1998; McClanahan & Hansen 2005; Jensen et al. 2010; Connelly & Brown 2011). Notably, the MS has been used annually to estimate the recreational catch of spiny lobster in Florida since 1991 (Sharp et al. 2005); brown trout in Tasmania since 1985 (Davies 1995); and for many different species in Alaska since 1977 (Jennings et al. 2009). Whilst this research has been applied to the recreational western rock lobster fishery, the
presented theoretical estimators in this research can be applied in these and other fisheries.

The identification of estimators such as $\hat{\gamma}_m$ and $\hat{\gamma}_e$ to estimate the mean value of dichotomous variables such as $p_{t,v}$ and $r_{t,v}$ is a step towards reducing the need of running more accurate but more expensive survey methods to calibrate the MS. This research also provides an alternative to using weighting methods that have the burden of identifying all important auxiliary data to successfully adjust for nonresponse bias. Further work however, is necessary to reduce the impact of recall bias in the reported days fished which is a barrier to improved estimates of the proportion of fishers who are avid, and leads to over estimates in the average retained catch per fisher. Improvements in this area could lead to total catch estimates from a MS using $\hat{\gamma}_e$ or $\hat{\gamma}_m$ that are very similar to those of the generally considered reliable, but more expensive, PDS.
References


References


References


References


References


References


Every reasonable effort has been made to acknowledge the owners of copyright material. I would be pleased to hear from any copyright owner who has been omitted or incorrectly acknowledged.
Appendix A

The mail survey form used for the 1995/96 end of season mail recall survey of people licensed to fish recreationally for western rock lobster and is similar to those survey forms used for seasons 1986/87 – 1997/98.
Appendix B

The mail survey form used for the 1999/2000 end of season mail recall survey of people licensed to fish recreationally for western rock lobster and, with exception to a few question variations over time, is largely representative of all such survey forms for seasons 1998/99 – present.

![Survey Form Image]
Appendix B

## Where did you do most of your fishing? (Please refer to the table and map below and list map code number for each locality or town with [1] being the most often fished). Please note the number of days fished using each method.

<table>
<thead>
<tr>
<th>[1] Town/Locality</th>
<th>Map code no.</th>
<th>Days fished at each locality</th>
<th>Pots</th>
<th>Diving</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2] Town/Locality</td>
<td>Map code no.</td>
<td>Days fished at each locality</td>
<td>Pots</td>
<td>Diving</td>
<td>Other</td>
</tr>
<tr>
<td>[3] Town/Locality</td>
<td>Map code no.</td>
<td>Days fished at each locality</td>
<td>Pots</td>
<td>Diving</td>
<td>Other</td>
</tr>
</tbody>
</table>

### Map code

<table>
<thead>
<tr>
<th>Map code</th>
<th>Area Description</th>
<th>Map code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Esperance to just east of Bremer Bay</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>Bremer Bay to just east of Walpole</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>Walpole to just south of Cape Leeuwin</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>Cape Leeuwin to Cape Naturaliste</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>Busselton Area</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>Bunbury to just south of Mandurah</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Mandurah</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>Rottnest</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>Rockingham</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>Metropolitan</td>
<td></td>
</tr>
</tbody>
</table>

---

**Enlargement**

**Western Australia**

---

Thank you for taking the time to complete this survey.
Appendix B
Appendix C

Determining $E[\Pi_x|\Pi_x > \Pi_{\bar{x}}]$ and $E[\Pi_{\bar{x}}|\Pi_x > \Pi_{\bar{x}}]$ when $\Pi_i \sim U(g_i, 1), i = \bar{x}, x$.

Assuming that the response parameters are uniformly distributed over their respective Manski’s Bound (i.e. $\Pi_i \sim U(g_i, 1)$ where $g_i = \frac{\bar{x}_i}{w_i}$ for $i = \bar{x}, x$, and $\bar{x} = x$), the calculation of values for $E[\Pi_x|\Pi_x \geq \Pi_{\bar{x}}]$ and $E[\Pi_{\bar{x}}|\Pi_x \geq \Pi_{\bar{x}}]$ depend on the comparative magnitude of the lower bounds on $\Pi_x$ and $\Pi_{\bar{x}}$:

Case i) Lower bound on $\Pi_x$ is at least the magnitude for that on $\Pi_{\bar{x}}$ i.e. $g_{\bar{x}} \geq g_x$ (Figure C.1):

![Diagram](image)

**Figure C.1**: The feasible region ($A$) for $\pi_x$ and $\pi_{\bar{x}}$ when $\pi_x > \pi_{\bar{x}}$ and $g_{\bar{x}} \geq g_x$, where $\Pi_i \sim U(g_i, 1), i = \bar{x}, x$. The area of $A$ is $|A| = \frac{(1-g_{\bar{x}})^2}{2}$. The range of possible $\pi_{\bar{x}}$ values for a given $\pi_x$ is represented by line segment $t$ which has length $|t| = \pi_x - g_{\bar{x}}$. The range of possible $\pi_x$ values for a given $\pi_{\bar{x}}$ is represented by line segment $s$ which has length $|s| = 1 - \pi_{\bar{x}}$. 

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Appendix C

Referring to Figure C.1, the domain for $\pi_x$ and $\pi_\bar{x}$ is constructed such that $\Pi_i \sim U(g_i, 1), i = \bar{x}, x$ and $\pi_x > \pi_\bar{x}$, and it is seen that the marginal densities for $\pi_x$ and $\pi_\bar{x}$ are $|t|/|A|$ and $|s|/|A|$, respectively. More formally, these density functions are:

$$f(\pi_x | \pi_x > \pi_\bar{x}) = \frac{\pi_x - g_\bar{x}}{|A|}, \quad g_\bar{x} \leq \pi_x \leq 1$$

and

$$g(\pi_\bar{x} | \pi_x > \pi_\bar{x}) = \frac{1 - \pi_x}{|A|}, \quad g_\bar{x} \leq \pi_\bar{x} \leq 1$$

where $|A| = \frac{(1-g_x)^2}{2}$.

Therefore,

$$E[\Pi_x | \Pi_x > \Pi_\bar{x}] = \int_{g_\bar{x}}^{1} \frac{\pi_x - g_\bar{x}}{|A|} d\pi_x$$

$$= \frac{1}{|A|} \left( \frac{\pi_x^3}{3} - \frac{g_\bar{x} \pi_x^2}{2} \right)_{g_\bar{x}}$$

$$= \frac{1}{|A|} \left( \frac{1}{3} - \frac{g_\bar{x}}{2} - \frac{g_\bar{x}^3}{3} + \frac{g_\bar{x}^3}{2} \right)$$

$$= \frac{1}{|A|} \left( \frac{1}{3} - \frac{g_\bar{x}}{2} + \frac{g_\bar{x}^3}{6} \right)$$

and

$$E[\Pi_\bar{x} | \Pi_x > \Pi_\bar{x}] = \int_{g_\bar{x}}^{1} \frac{1 - \pi_\bar{x}}{|A|} d\pi_\bar{x}$$

$$= \frac{1}{|A|} \left( \frac{\pi_\bar{x}^3}{2} - \frac{\pi_\bar{x}^2}{3} \right)_{g_\bar{x}}$$

$$= \frac{1}{|A|} \left( \frac{1}{2} - \frac{1}{3} - \frac{g_\bar{x}^2}{2} + \frac{g_\bar{x}^3}{3} \right)$$

$$= \frac{1}{|A|} \left( \frac{1}{6} - \frac{g_\bar{x}^2}{2} + \frac{g_\bar{x}^3}{3} \right)$$

where $|A| = \frac{(1-g_x)^2}{2}$.
Case ii) Lower bound on $\Pi_x$ is smaller in magnitude than that on $\Pi_{\bar{x}}$, i.e. $q_{\bar{x}} < q_x$ (Figure C.2):

![Figure C.2: The feasible region (B) for $\pi_x$ and $\pi_{\bar{x}}$ when $\pi_x > \pi_{\bar{x}}$ and $q_{\bar{x}} < q_x$, where $\Pi_i \sim U(g_i, 1), i = \bar{x}, x$. The area of $B$ is $|B| = (1 - g_x)(g_x - g_{\bar{x}}) + \frac{(1 - g_x)^2}{2}$. The range of possible $\pi_{\bar{x}}$ values for a given $\pi_x$ is represented by line segment $t$ which has length $|t| = \pi_x - g_{\bar{x}}$. The range of possible $\pi_x$ values for a given $\pi_{\bar{x}}$ is represented by line segment $s_1$ if $q_{\bar{x}} \leq \pi_{\bar{x}} \leq g_x$ or $s_2$ if $g_x \leq \pi_{\bar{x}} \leq 1$. The length of $s_1$ is $|s_1| = 1 - g_x$ and the length of $s_2$ is $|s_2| = 1 - \pi_{\bar{x}}$.

Referring to Figure C.2, the domain for $\pi_x$ and $\pi_{\bar{x}}$ is constructed such that $\Pi_i \sim U(g_i, 1), i = \bar{x}, x$ and $\pi_x > \pi_{\bar{x}}$, and it is seen that the marginal densities for $\pi_x$ and $\pi_{\bar{x}}$ are $|t|/|B|$ and $I(g_{\bar{x}} \leq \pi_{\bar{x}} \leq g_x)|s_1|/|B| + I(g_x \leq \pi_{\bar{x}} \leq 1)|s_2|/|B|$, respectively, where $I(z)$ is the identify function and is 1 if condition $z$ is true and 0 if false. More formally, these density functions are:
Appendix C

\[ f(\pi_x | \pi_x > \pi_x) = \frac{\pi_x - g_{\pi}}{|B|}, \quad g_x \leq \pi_x \leq 1 \]

and

\[ g(\pi_x | \pi_x > \pi_x) = \begin{cases} \frac{1 - g_{\pi}}{|B|} & g_x \leq \pi_x \leq g_x \\ \frac{1 - \pi_x}{|B|} & g_x \leq \pi_x \leq 1 \end{cases} \]

where \(|B| = (1 - g_x)(g_x - g_{\pi}) + \frac{(1 - g_{\pi})^2}{2}.

Therefore,

\[ E[\Pi_x | \Pi_x > \Pi_x] = \int_{g_x}^{1} \frac{\pi_x - g_{\pi}}{|B|} d\pi_x \]

\[ = \frac{1}{|B|} \left( \frac{\pi_x^3}{3} - \frac{g_{\pi} \pi_x^2}{2} \right) \bigg|_{g_x}^{1} \]

\[ = \frac{1}{|B|} \left( \frac{1}{3} - \frac{g_{\pi}}{2} - \frac{g_x^3}{3} + \frac{g_x g_{\pi}^2}{2} \right) \]

\[ = \frac{1}{|B|} \left( \frac{1}{3} - \frac{g_{\pi}}{2} - g_x^2 \left( \frac{g_x}{3} - \frac{g_{\pi}}{2} \right) \right) \]

and

\[ E[\Pi_x | \Pi_x > \Pi_x] = \int_{g_x}^{g_{\pi}} \frac{1 - g_x}{|B|} d\pi_x + \int_{g_x}^{1} \frac{1 - \pi_x}{|B|} d\pi_x \]

\[ = \frac{1}{|B|} \left( \frac{1 - g_x}{2} \pi_x^2 \bigg|_{g_x}^{g_{\pi}} + \frac{1}{|B|} \left( \frac{\pi_x^3}{3} \bigg|_{g_x}^{1} - \pi_x \bigg|_{g_x}^{g_{\pi}} \right) \right) \]

\[ = \frac{1}{|B|} \left( \frac{(1 - g_x)(g_{\pi}^2 - g_x^2)}{2} + \frac{1}{2} - \frac{1}{3} - \frac{g_x^3}{2} + \frac{g_{\pi}^3}{3} \right) \]

\[ = \frac{1}{|B|} \left( \frac{g_{\pi}^2 - g_x^2 - g_x^3 + g_x g_{\pi}^2}{2} + \frac{1}{6} - \frac{g_x^3}{2} + \frac{g_{\pi}^3}{3} \right) \]

\[ = \frac{1}{|B|} \left( \frac{-g_{\pi}^2 - g_x^3 + g_x g_{\pi}^2}{2} + \frac{1}{6} + \frac{g_{\pi}^3}{3} \right) \]

\[ = \frac{1}{|B|} \left( \frac{1}{6} - \frac{g_{\pi}^2(1 - g_x)}{2} - \frac{g_x^3}{6} \right) \]

where \(|B| = (1 - g_x)(g_x - g_{\pi}) + \frac{(1 - g_{\pi})^2}{2}.

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Appendix D

R-code used to generate 1,000,000 random samples of \( n_1 \) and \( n_0 \) for a specific value of \( p, \pi_1 \) and \( \pi_0 \):

\[
sims<-1000000; \quad \text{N}<40000; \quad \text{n}<40000; \quad \text{p}<0.6; \quad \text{pi1}<0.7; \quad \text{pi0}<0.4
\]

\[
\text{set.seed(600)}
\]

\[
\text{N1}<\text{round(}N*\text{p*pi1,0)} \quad \# \text{fish will return survey}
\]

\[
\text{N0}<\text{round(}N*(1-p)*\text{pi0,0)} \quad \# \text{do not fish but will return survey}
\]

\[
\text{min.fish}<\text{round(max(c(1,n-round(}N*(1-p),0)),0)}
\]

\[
\text{max.fish}<\text{round(min(c(n-1,round(}N*\text{p)),0)}
\]

\# COMMENT: \text{pbinom} is equivalent to \text{Cbinom}

\[
\text{low.cd}<\text{pbinom(min.fish,n,p)}
\]

\[
\text{up.cd}<\text{pbinom(max.fish,n,p)}
\]

\[
\text{drand}<\text{runif(sims,low.cd,up.cd)}
\]

\[
\text{fishers}<\text{as.integer(qbinom(drand,n,p))}
\]

\[
\text{Matrix <- matrix(c(fishers-N*p*(1-pi1),rep(1,length(fishers))), nrow=sims, ncol=2)}
\]

\[
\text{min.n1 <- apply(Matrix, 1, max)}
\]

\[
\text{Matrix <- matrix(c(fishers,rep(N*p*pi1,length(fishers))), nrow=sims, ncol=2)}
\]

\[
\text{max.n1 <- apply(Matrix, 1, min)}
\]

\[
\text{low.cd}<\text{pbinom(min.n1,fishers,pi1)}
\]

\[
\text{up.cd}<\text{pbinom(max.n1,fishers,pi1)}
\]

\[
\text{drand}<\text{runif(sims,low.cd,up.cd)}
\]

\[
\text{all.n1}<\text{as.integer(qbinom(drand,round(fishers,0),pi1))}
\]

\[
\text{nfishers}<\text{n-fishers}
\]

\[
\text{Matrix <- matrix(c(nfishers-N*(1-p)*(1-pi0),rep(1,length(nfishers))), nrow=sims, ncol=2)}
\]

\[
\text{min.n0 <- apply(Matrix, 1, max)}
\]

\[
\text{Matrix <- matrix(c(nfishers,rep(N*(1-p)*pi0,length(nfishers))), nrow=sims, ncol=2)}
\]

\[
\text{max.n0 <- apply(Matrix, 1, min)}
\]

\[
\text{low.cd}<\text{pbinom(min.n0,nfishers,pi0)}
\]

\[
\text{up.cd}<\text{pbinom(max.n0,nfishers,pi0)}
\]

\[
\text{drand}<\text{runif(sims,low.cd,up.cd)}
\]

\[
\text{dall.n0}<\text{as.integer(qbinom(drand,nfishers,pi0))}
\]
Appendix E

S-Plus (version 8.0.4, Insightful Corp.) code used for identifying the optimal time series model to describe $\hat{L}_{t,v,y_0}$ for each fishing method $\nu \in \{\text{pot}, \text{dive}\}$ and estimator $y_0 \in \{\tilde{y}_0, \tilde{y}_m\}$.

```
# Code used for identifying optimal time series model for
# licence usage (Lt) for each fishing method (potting, diving)
# using either po or pm to estimate the proportion of licences used.
#
AICc_nofit<-function(y){
  # models will also be compared to that
  # of just estimating the series y by its mean.
  x<-lm(y~1)
  npar<-length(coef(x))
  nobs<-length(y)
  return(AIC(x)+2*npar*(npar+1)/(nobs-npar-1))
}

AICc<-function(x){
  # models will be compared based on the AIC with a correction.
  npar<-nrow(x$var.coef)+length(x$reg.coef)
  nobs<-x$n.used
  return(x$aic+2*npar*(npar+1)/(nobs-npar-1))
}

# select the fishing method for which we are modelling
# the licence usage number
dmetho<-c("Potting","Diving")[1]

ewDat<-newDatPotDive[newDatPotDive$fishmeth==dmetho,]
newDat$Season<-as.character(newDat$Season)
newDat$po<-newDat$n1/(newDat$n0+newDat$n1)
newDat$poFishers<-(newDat$N*newDat$po)

# select the estimator for which we want to model the
# resulting estimates for licence usage for fishing method
# "dmetho".

g<-c("ratio","multi")[1]
if (g=="ratio"){
```
newDat$Usage<-log(newDat$poFishers)
} else{
  newDat$Usage<-log(newDat$pmFishers)
}
newDat2<-newDat[!is.na(newDat$Usage),]

# The maximum number of points that can be used by
# any time series model given the last 8 seasons are
# withheld for assessing model forecasting ability in
# further analysis'.
est.pts<-nrow(newDat2)-8

# SELECT TIME SERIES MODEL (no regressors...yet!)
# Fit ARMA(1,0,0) model for usage
# "spoint" is adjusted upwards for models with a lower order
# to account for the extra data points used to condition the
# model for the highest ordered model considered remembering
# that AICc is only comparable between models that model the
# same number of sample points.
spoint<-2
fitmodel<-list(order=c(1,0,0))
fit100<-arima.mle(newDat2$Usage[spoint:est.pts]-
mean(newDat2$Usage[spoint:est.pts]),fitmodel,
max.fcal=1000,maxiter=1E100)
AICc(fit100)
# estimate roots of the characteristic equation for
# determining stationary and invertibility conditions.
abs(polyroot(c(1,-fit100$model$ar)))
# need to determine the AICc of the model that does
# not fit a time series model at all: reduce the number
# of modelled points by one i.e. "spoint+1".
AICc_nofit(newDat2$Usage[(spoint+1):est.pts])

# Fit ARMA(0,0,1) model for usage
# spoint<-2
fitmodel<-list(order=c(0,0,1))
fit001<-arima.mle(newDat2$Usage[spoint:est.pts]-
mean(newDat2$Usage[spoint:est.pts]),fitmodel,max.fcal=1000,maxiter=1E100)
AICc(fit001)
abs(polyroot(c(1,-fit001$model$ma)))
Appendix E

# Fit ARMA(1,0,1) model for usage

```r
spoint<-1
fitmodel<-list(order=c(1,0,1))
fit101<-arima.mle(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),fitmodel,max.fcal=1000,maxiter=1E100)
AICc(fit101)
abs(polyroot(c(1,-fit101$model$ar)))
abs(polyroot(c(1,-fit101$model$ma)))
```

# Fit ARMA(2,0,0) model for usage

```r
spoint<-1
fitmodel<-list(order=c(2,0,0))
fit200<-arima.mle(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),fitmodel,max.fcal=1000,maxiter=1E100)
AICc(fit200)
abs(polyroot(c(1,-fit200$model$ar)))
```

# ARMA(1,0) best for both pot, dive and po,pm. Now investigate regressors.

```r
spoint<-1

# AR(1) but NO regressor

```r
fitmodel<-list(order=c(1,0,0))
fit100<-arima.mle(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),fitmodel,max.fcal=1000,maxiter=1E100)
AICc(fit100)
abs(polyroot(c(1,-fit100$model$ar)))
```

# Include lnAlk3 as regressor

```r
newDat2$puer<-newDat2$lnAlk3
fitmodel<-list(order=c(1,0,0))
fit100<-arima.mle(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),fitmodel,
xreg=cbind(newDat2$puer-mean(newDat2$puer[spoint:est.pts])[spoint:est.pts],[spoint:est.pts],max.fcal=1000,maxiter=1E100)
```
Appendix E

```r
all.reg<-cbind(newDat2$puer-mean(newDat2$puer[spoint:est.pts])),[spoint:est.pts,]
fitted<-arima.filt(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),model=fit100$model,
xreg=all.reg,reg.coef=fit100$reg.coef)
xresid<-(newDat2$puer-mean(newDat2$puer[spoint:est.pts]))[spoint:est.pts]-fitted$pred
AICc(fit100)
abs(polyroot(c(1,-fit100$model$ar)))

# Include lnAlk4 as regressor

newDat2$puer<-newDat2$lnAlk4
fitmodel<-list(order=c(1,0,0))
fit100<-arima.mle(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),fitmodel,
xreg=cbind(newDat2$puer-mean(newDat2$puer[spoint:est.pts]))[spoint:est.pts],max.fcal=1000,maxiter=1E100)
all.reg<-cbind(newDat2$puer-mean(newDat2$puer[spoint:est.pts]))[spoint:est.pts,]
fitted<-arima.filt(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),model=fit100$model,
xreg=all.reg,reg.coef=fit100$reg.coef)
AICc(fit100)
abs(polyroot(c(1,-fit100$model$ar)))

# Include lnAlk3 & lnAlk4 as regressors

newDat2$puer1<-newDat2$lnAlk3
newDat2$puer2<-newDat2$lnAlk4
fitmodel<-list(order=c(1,0,0))
fit100<-arima.mle(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),fitmodel,
xreg=cbind(newDat2$puer1-mean(newDat2$puer1[spoint:est.pts]),newDat2$puer2-mean(newDat2$puer2[spoint:est.pts]))[spoint:est.pts],,
max.fcal=1000,maxiter=1E100)
all.reg<-cbind(newDat2$puer1-mean(newDat2$puer1[spoint:est.pts]),newDat2$puer2-
mean(newDat2$puer2[spoint:est.pts]))[spoint:est.pts],)
fitted<-arima.filt(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),model=fit100$model,
xreg=all.reg,reg.coef=fit100$reg.coef)
AICc(fit100)
abs(polyroot(c(1,-fit100$model$ar)))
```
# Get parameter estimates and their s.e.'s for the identified optimal models of licence usage.

# POTTING - optimal model includes lnAlk4 as a regression variable.

spoint<-1
npars<-2
newDat2$puer<-newDat2$lnAlk4
fitmodel<-list(order=c(1,0,0))
fit<-arima.mle(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),fitmodel,
  xreg=cbind(newDat2$puer-mean(newDat2$puer[spoint:est.pts]))[spoint:est.pts,],max.fcal=1000,maxiter=1E100)
fit$sigma2
fitted<-arima.filt(newDat2$Usage[spoint:est.pts]-mean(newDat2$Usage[spoint:est.pts]),model=fit$model,
  xreg=cbind(newDat2$puer-mean(newDat2$puer[spoint:est.pts]))[spoint:est.pts,],reg.coef=fit$reg.coef)
regfilt2<-arima.filt(newDat2$puer[1:nrow(newDat2)]-mean(newDat2$puer[1:nrow(newDat2)]),fit$model)
regfilt<-cbind(regfilt2$ filt)
regpred<-cbind(regfilt2$ pred)
W<-as.matrix(regfilt)[(fit$n.cond+1):est.pts,]-as.matrix(regpred)[(fit$n.cond+1):est.pts,]
Xhess<-solve(t(W) %*% W)
# regression parameter
fit$reg.coef
# s.e. of regression parameter
sqrt(fit$sigma2*diag(Xhess))
# AR parameter
(fit$model)$ar
# s.e. of AR parameter
sqrt(fit$var.coef)

# DIVING - optimal model does not include a regression variable.

spoint<-1
npars<-1
fitmodel<-list(order=c(1,0,0))
fit<-arima.mle(newDat2$Usage[spoint:est.pts]-
mean(newDat2$Usage[spoint:est.pts]),fitmodel,max.fcals=1000,maxiter=1E100)
# AR parameter
(fit$model)$ar
# s.e. of AR parameter
sqrt(fit$var.coef)
Appendix F

The ACF and PACF plots for the number of licensees estimated to have fished each season $t$ using fishing methods $v \in \{\text{pot}, \text{dive}\}$ ($\hat{L}_{t,v,y_p}^{(m)}$) where $y_p = \hat{\gamma}_m$ is calculated using the mail survey data.

![Image of ACF and PACF plots](image)

**Figure F.1**: Autocorrelations (ACF) of $\ln \hat{L}_{t,v,y_p}^{(m)}$ for each lag where $\hat{L}_{t,v,y_p}^{(m)}$ is the number of potters estimated by applying $\hat{\gamma}_m$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).
Figure F.2: Partial autocorrelations (PACF) of $\ln \hat{I}_{t,e,y_p}^{(m)}$ for each lag where $\hat{I}_{t,e,y_p}^{(m)}$ is the number of potters estimated by applying $\hat{\gamma}_m$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).

Figure F.3: Autocorrelations (ACF) of $\ln \hat{I}_{t,e,y_p}^{(m)}$ for each lag where $\hat{I}_{t,e,y_p}^{(m)}$ is the number of divers estimated by applying $\hat{\gamma}_m$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).
Appendix F

Figure F.4: Partial autocorrelations (PACF) of $\ln \hat{L}^{(m)}_{t,v,w_0}$ for each lag where $\hat{L}^{(m)}_{t,v,w_0}$ is the number of divers estimated by applying $\hat{\gamma}_m$ to the mail survey data. The 95% confidence interval for a zero(0) correlation is also included (dashed lines).